2009 Algebra Applications Topic Test

Solutions:

1. $s_a = \dfrac{k}{\sqrt{d}} \qquad k = s_a\sqrt{d} \quad s_b = \dfrac{s_a\sqrt{d}}{\sqrt{.25d}} = s_a\sqrt{\dfrac{1}{.25}} = s_a\sqrt{4} = 2s_a$ An increase of 100%

2. First firm price = $(80)(.8)(.9) = \$57.60$ Second firm price= $(80)(.7) = \$56.00$

   The second firm is a better price by $1.60

3. If Adam buys x balls, each costing $y, then x balls with 5% tax equals x+3 balls without tax

   $xy(1.05) = (x+3)y \Rightarrow (1.05)x = x+3 \Rightarrow (.05)x = 3 \Rightarrow x = 60$

4. Let the original number of boys = $5x$ Then: $\dfrac{5x+24}{7x} = \dfrac{7}{5}$ and

   and the original number of girls = $7x$ $49x = 25x+120$ and $x = 5$ and $7x = 35$

5. Let $x =$ amount removed and replaced. Then $5(.25) - x(.25) + x(.75) = 5(.55)$

   and $.5x = 1.5$, and $x = 3$

6. Rate $*$ Time = Distance, $1.5a + 1.5b = 120$ and $2a + b = 120$

   Solving $a = 40$ and $b = 40$

7. Let $t =$ the time for each train rate $*$ time = distance $40t + 40t = 120 \Rightarrow$

   $t = \dfrac{3}{2}$ hours The trains meet after $\dfrac{3}{2}$ hours, and the pigeon's distance is $60\left(\dfrac{3}{2}\right) =$ 90 miles

8. A = 94_{10} B = 92_{10} C = 96_{10} D = 101_{10} and the correct order is BACD.

9. Let T = points already earned on x tests

   $\dfrac{T+97}{x+1} = 90 \Rightarrow T+97 = 90x+90 \Rightarrow T = 90x-7$

   $\dfrac{T+73}{x+1} = 87 \Rightarrow T+73 = 87x+87 \Rightarrow T = 87x+14$

   $90x-7 = 87x+14 \Rightarrow 3x=21 \Rightarrow x = 7$

10. For n>4, the units digit of n! is 0. The units digit of the given sum is therefore determined by

    1! + 2! + 3! + 4! =1 + 2 + 6 + 24 = 33. so $x = 3$.

    $2^3 = 8$, $\begin{vmatrix} 4 & 3 \\ 2 & 5 \end{vmatrix} = 20 - 6 = 14$, ${}_5C_3 =$ 10, therefore the sum is 8+14+10=32

11. T = time of 1st bus in hours $T - \dfrac{1}{5} =$ time of 2nd bus in hours and distances are the same

    $50T = 55\left(T - \dfrac{1}{5}\right) \Rightarrow T = \dfrac{11}{5}$ hours or 2 hours and 12 minutes, $\therefore$ time was 5:12 p.m.

12. X = the least common multiple of (3, 4, 7, 13) + 1 = 3*5*7*13 + 1 = 1366

13 Let the three numbers be x, y, and z. Then, $x + y + z = 88$, $x - 5 = y + 5 \Rightarrow$
$x - y = 10$, combining these two equations gives $2x + z = 98$, combining $x - 5 = 5z$
with 2x+z=98 gives 11z=88 or z=8 $\Rightarrow$ x = 45 and y = 35 . $45 - 8 = 37$

14 By the Remainder Theorem:

$$\frac{f(x)}{x+1} = (-1)^5 - 2(-1)^4 + a(-1)^3 - (-1)^2 + b(-1) - 2 = -7 \Rightarrow a + b = 1$$

$$\frac{f(x)}{x-2} = 2^5 - 2(2)^4 + a(2)^3 - (2)^2 + b(2) - 2 = 32 \Rightarrow 4a + b = 19,$$

solving these simultaneous equations gives $a = 6$ and $b = -5$

$$\frac{f(x)}{x-1} = 1^5 - 2(1)^4 + 6(1)^3 - (1)^2 - 5(1) - 2 = -3$$

15. $798 \ \Theta \ 1211 = 799 + 800 + 801 + ... + 1209 + 1210$

$799 \ \Theta \ 1210 = 800 + 801 + 802 + ... + 1209$

$(798 \ \Theta \ 1211) - (799 \ \Theta \ 1210) = 799 + 1210 = 2009$

16. The graph is a circle.

$$(x^2 - 6x + 9) + (y^2 + 8y + 16) = -13 + 9 + 16 = 12 \Rightarrow (x-3)^2 + (y+4)^2 = 12$$

$r^2 = 12$ Area = 12π

17. The series is $S = 35 + 42 + 49 + ...4998$, it is arithmetic with $d = 7$. $a_n = a_1 + (n-1)d$

$4998 = 35 + (n-1)7 \Rightarrow 4963 = (n-1)7 \Rightarrow 710 = n$ $\therefore$ the series has 710 terms

$$S_{710} = \frac{710}{2}(35 + 4998) = 1,786,715$$

18. $I = \dfrac{k}{d^2} \Rightarrow k = Id^2$, $\quad 12(4)^2 = I(8)^2 \Rightarrow 192 = 64I \Rightarrow I = 3$

19. $(2 * 36) = \sqrt{36} + \dfrac{36}{2} = 6 + 18 = 24$, $\quad (3 * 27) = \sqrt[3]{27} + \dfrac{27}{3} = 3 + 9 = 12$

$$24 \Psi 12 = (24 + 12)^{\frac{12}{24}} = (36)^{\frac{1}{2}} = 6$$

20. Let x be the original salary

$$\frac{1}{3}x + \frac{1}{3}x + \frac{1}{2}x + \left(\frac{1}{20}\right)\left(\frac{1}{3}\right)x + 450 = 2\left(\frac{2}{3}x\right) \Rightarrow \frac{70}{60}x + \frac{1}{60}x + 450 = \frac{80}{60}x \Rightarrow$$

$$\frac{9}{60}x = 450 \Rightarrow x = \left(\frac{60}{9}\right)(450) = 3000$$

21. Let x = the original number of students and $\frac{\mathrm{B}}{\mathrm{x}}$ = y = the original cost per student and B = xy.

$$\frac{\frac{xy}{2}}{x-5}=y-\frac{1}{2},\quad \frac{xy}{x-5}=2y-1,\quad xy=2xy-10y-x+5,\ \ 10y-xy=5-x,\ \ y=\frac{5-x}{10-x}$$

$$\frac{\frac{6}{5}xy}{x+10}=y-\frac{3}{10},\quad \frac{12xy}{x+10}=10y-3,\quad 12xy=10xy-3x+100y-30,\ \ 2xy-100y=-3x-30$$

$y=\frac{-3x-30}{2x-100}$, Solving together, $\frac{5-x}{10-x}=\frac{-3x-30}{2x-100}\ \Rightarrow\ 5x^2-110x+200=0\ \Rightarrow$

$x^2-22x+40=0\ \Rightarrow\ (x-2)(x-20)=0$ only x = 20 is usable

22. $x+y+z=32,\ \ \frac{\mathrm{y}}{\mathrm{x}}=4,\ \ z=\frac{3}{5}(x+y)\ \Rightarrow\ y=4x,\ \ z=\frac{3}{5}(x+4x)=3x$

$x+4x+3x=8x=32\ \Rightarrow\ x=4\ \Rightarrow\ z=3x=12$

23. $6x+8y=500\ \Rightarrow\ y=\frac{500}{8}-\frac{6x}{8}\ \Rightarrow\ y=\frac{250}{4}-\frac{3x}{4}$, values of x must fall $0<x<83\frac{1}{3}$,

y will have integer values only if x is divisible by 2 but not by 4. $\frac{83}{2}=41+$ and $\frac{83}{4}=20+\ \Rightarrow$

There are $41-20=21$ combinations possible.

24. a, b, and c are rates for valves A, B, and C. $1(a+b+c)=1,\ \frac{3}{2}(a+c)=1,$

$2(b+c)=1\ \Rightarrow\ a+c=\frac{2}{3},\ b+c=\frac{1}{2}\ \Rightarrow\ a+\frac{1}{2}=1,\ a=\frac{1}{2}\ \Rightarrow\ c=\frac{2}{3}-\frac{1}{2}=\frac{1}{6}\ \Rightarrow$

$b+\frac{1}{6}=\frac{1}{2}\ \Rightarrow\ b=\frac{1}{3},\ \ a+b=\frac{1}{2}+\frac{1}{3}=\frac{5}{6},\ \ t\left(\frac{5}{6}\right)=1\ \Rightarrow\ t=\frac{6}{5}=1.2$ houurs

25. If her meal's cost before tax and tip is \$x, then his meal's cost is \$(x+1).

$.15(1.06(x+1))=.16x\ \Rightarrow\ 15(1.06(x+1))=16x\ \Rightarrow\ 15.9x+15.9=16x\ \Rightarrow$

$.1x=15.9\ \Rightarrow\ x=\$159$

26. Let $\mathrm{N}=3.2513513513...\ \Rightarrow\ 10000\mathrm{N}=32513.513513...$ and

$-10\mathrm{N}=32.513513513...\ \Rightarrow\ 9990\mathrm{N}=32481\ \Rightarrow\ \mathrm{N}=\frac{32481}{9990}=\frac{3609}{1110}$

$\mathrm{N}=\frac{1203}{370}$

27. For each of the 9 legs of the trip, the cat travels the same distance $d=rt=$

$\frac{miles}{hr}*\min utes*\frac{1hr}{60\min utes}$. For the first section distance is $\frac{1}{2}*8*\frac{1}{60}=\frac{4}{60}\,miles$

and the total distance = $9*\frac{4}{60}=\frac{36}{60}=\frac{6}{10}=.6$ miles

28. $\mathrm{F} = \frac{5}{9}(\mathrm{F}+20-32) \;\Rightarrow\; \mathrm{F} = -15$

29. Let h = the required height of juice and volume of juice + 10% of volume of juice = volume of container. $\pi(12)^2 h + \frac{1}{10}\pi(12)^2 h = \pi(12)^2(16.5) \;\Rightarrow\; h + \frac{1}{10}h = 16.5 \;\Rightarrow$ $\frac{11}{10}h = 16.5 \;\Rightarrow\; h = \frac{10}{11}(16.5) = 15$

30. Let P be the amount of money to be distributed. The first and second born children's monies are equal.

$$1000 + \frac{1}{10}(P-1000) = 2000 + \frac{1}{10}\left(P - 2000 - \left(1000 + \frac{1}{10}(P-1000)\right)\right)$$

$$10000 + P - 1000 = 20000 + P - 2000 - 1000 - \frac{1}{10}P + 100$$

$\frac{1}{10}P = 8100 \;\Rightarrow\; P = 81000 \Rightarrow$ The first born child receives $1000 + \frac{1}{10}(80000) = 9000$

The second born child receives $2000 + \frac{1}{10}\left(81000 - 2000 - \left(1000 + \frac{1}{10}(81000-1000)\right)\right) =$

$2000 + \frac{1}{10}(79000 - 1000 - 8000) = 2000 + \frac{1}{10}(70000) = 9000.$ Each of the children inherits the same amount. $\therefore$ There are $\frac{81000}{9000} = 9$

Tiebreaker 1. New production = $(100+n)\%$ of $m = \left(\frac{100+n}{100}\right) * m = m\left(1 + \frac{n}{100}\right)$

Tiebreaker 2. There are nine 2-digit palindromic numbers and 90 three digit palindromic numbers. The number of 3-digit numbers can be found by 9 possibilities for the first digit and 10 for the second digit. There are $9*10=90$ such numbers.

Tiebreaker 3. Let us number the days of the week from Day 1 to Day 7. It will require 14 different calendars to cover the seven different starting days for non-leap years and leap years. A cycle of four years contains (366 + 3*365) days or 208 weeks and 5 days. If 1972 (the first leap year in the collection) starts on Day 1 then the starting days of subsequent leap years are:1976 - Day 6, 1980 - Day 4, 1984 - Day 2, 1988 - Day 7, 1992 - Day 5, 1996 - Day 3. The non-leap year calendars are collected much sooner. One way to cosider it is to note that in the years preceeding the leap years would provide all seven calendars. For example, 19 71 - Day 7, 1975 - Day 5, 1979 - Day 3, ...1995 - Day 2. $\therefore$ The collection is complete in 1996.