1. \( m\angle D = 30^\circ \) \( m\angle AED = 90^\circ \)
   
   \[ D = 2(AE) = 2r \quad BD = 3r \]
   
   \[ 6^2 = 3r(r) = 3r^2, \quad r = 2\sqrt{3}, \quad 3r = 6\sqrt{3} \]

2. \( DF = EF = 15 \quad CF = x, \quad BF = 34 - x \)
   
   \[ 15^2 = x(34 - x) \quad x^2 - 34x + 225 = 0 \]
   
   \[ (x-9)(x-25) = 0 \quad \text{smallest is } x = 9 \]

3. By the Pythagorean Theorem, \( BC = 4\sqrt{2} \).
   
   Area of sector = \( \frac{90}{360} \pi (4)^2 = 4\pi \)
   
   Area of \( \square ABC = \frac{1}{2}(4)(4) = 8 \). Area of segment = \( 4\pi - 8 \)

4. \( CD = BE = 2 \quad AE = 5 \quad CE^2 = 144 \),
   
   \[ CE^2 + 5^2 = 13^2 \quad CE = BD = 12 \]
   
   \[ CE^2 + 25 = 169 \]

5. \( (x-4)^2 + (y+8)^2 = r^2 \)
   
   Therefore, \( 80 - r^2 = -20 \)
   
   \[ x^2 - 8x + 16 + y^2 + 16y + 64 = r^2 \quad r^2 = 100 \]
   
   \[ x^2 + y^2 - 8x + 16y + (80 - r^2) = 0 \quad r = 10 \]

6. \( y + 6y + 8y = 360 \quad m\angle C = \frac{1}{2}(6y - y) = \frac{1}{2}(5y) = \frac{5y}{2} = \frac{5}{2}(24) = 60 \)
   
   \[ 15y = 360 \quad y = 24 \]

7. Let \( CD = 8, \ AB = 12, \ AC = 30, \ AE = x, \ CE = 30 - x; \)
   
   \( \triangle ABE \sim \triangle CDE, \frac{12}{x} = \frac{8}{30-x} \), \( 20x = 360, \ x = 18, \ 30 - x = 12. \)

   Applying the Pythagorean Theorem:
   
   \( (BE)^2 + 144 = 324 \quad BE = 6\sqrt{5} \)
   
   \( 8^2 + (DE)^2 = 12^2 \quad DE = 4\sqrt{5} \), \( BD = 10\sqrt{5} \).

8. The five arcs are congruent and each measures \( \frac{360}{5} = 72 \).
   
   \( m\angle CEG = \frac{1}{2} m\angle FCE = \frac{1}{2}(3)(72) = 108 \quad m\angle ECF = \frac{1}{2} m\angle F = \frac{1}{2}(72) = 36 \quad m\angle CEG + m\angle ECF = 108 + 36 = 144 \)
9. Solution: Radii \( \overline{AB} \) and \( \overline{CD} \) are \( \perp \) to \( \overline{BD} \). Draw \( \overline{CE} \perp \overline{AB} \).

In 30, 60, 90 rt \( \triangle ACE \), \( CE = 4\sqrt{3} \), \( \therefore BD = 4\sqrt{3} \)

Area of rectangle \( CDBE = (2)(4\sqrt{3}) = 8\sqrt{3} \)

Area of triangle \( ACE = \frac{1}{2}(4)(4\sqrt{3}) = 8\sqrt{3} \)

Area of \( ABDC = 8\sqrt{3} + 8\sqrt{3} = 16\sqrt{3} \)

10. \((x - 12)x = 16(10)\)

\( x^2 - 12x - 160 = 0 \)

\((x - 20)(x + 8) = 0 \)

\( x = 20 \quad x = -8 \) is unusable

11. \( A_1 = (5\sqrt{2})^2 = 50 \quad \frac{x^2}{4} + x^2 = 25 \quad \frac{5x^2}{4} = 25 \quad x^2 = 20 = A_2 \)

\( \frac{A_1}{A_2} = \frac{50}{20} = \frac{5}{2} \)

12. Solution: \( FE = 11 \) because tangents to a circle from an exterior point are equal.

13. Since these are chords intersecting inside the circle,

\((x - 1)(7) = 3(x + 4), \quad 4x = 5, \quad x = \frac{5}{4} \).

14. Solution: \( m\angle D = \frac{1}{2}(160) = 80^\circ \), \( m\angle FEB = 80^\circ \), \( m\overline{BC} = m\overline{DE} = 120^\circ \),

\( m\overline{DC} = 360 - (120 + 120 + 40) = 80 \quad m\angle F = \frac{1}{2}(80 - 40) = 20 \)
15. Solution: Area of triangle = $\frac{1}{2} (9)(6\sqrt{3}) = 27\sqrt{3}$

16. $m\angle DBC = 90^\circ - 40^\circ = 50^\circ$, $\square DBC$ is isosceles, 
   $\therefore \angle D = 180^\circ - (50^\circ + 50^\circ) = 80^\circ$

17. Solution: $30 = \frac{1}{2} (150 - m\overline{BE})$, $60 = 150 - m\overline{BE}$, $m\overline{BE} = 90$, 
   $m\angle CGD = \frac{1}{2} (150 + 90) = \frac{1}{2} (240) = 120$

18. Solution: Since the angle between 2 tangents and the central angle are 
   supplementary, $m\angle BAC = 124^\circ$, and the inscribed angle, $m\angle BEC = 62^\circ$, 
   $\square BAE \cong \square CAE$, and $m\angle BEA = 31^\circ$, $\therefore$ by $\cong$ angles of a isosceles $\square$, 
   $m\angle ABE = 31^\circ$

19. Let the radius of the circumscribed circle be $r$, and the area of the circumscribed 
   circle is $\pi r^2$. The triangle is a 45-45-90 triangle and the radius of the inscribed 
   triangle is $\frac{r}{\sqrt{2}}$ and the area of the inscribed circle is $\frac{\pi r^2}{2}$. The ratio is $\frac{\pi r^2}{2} = \frac{2}{1}$

20. Solution: 
   $\angle BAD = 30^\circ$ by symmetry. Let E be the midpoint of AB. 
   DA and DB are radii of Circle B and are $\equiv$. 
   $\therefore$ $\square ABD$ is isosceles and $\square ADE$ is a 30, 60, 90 $\square$, 
   side opp. the $60^\circ \angle = \frac{1}{2}$ and the hypotenuse = $\frac{\sqrt{3}}{3}$.

21. These are rt. triangles, since they are inscribed in semicircles. The upper triangle has a 
   2nd leg of 13 by the Pythagorean theorem and the 2nd leg of the lower triangle is 11. The 
   combined areas are $\frac{1}{2}(1)(13) + \frac{1}{2}(7)(11) = \frac{13}{2} + \frac{77}{2} = \frac{90}{2} = 45$.

22. Let the radius of Circle A be $r$. $(15 + r)(15 - r) = 4(14)$, $225 - r^2 = 56$, 
   $r^2 = 169$, $r = 13$, $15 - 13 = 2$
23. \( y^2 = x^2 - 1, \quad y^2 = 12^2 - (x+1)^2, \quad x^2 - 1 = 12^2 - x^2 - 2x - 1 \)
\( 2x^2 + 2x - 144 = 0, \quad x^2 + x - 72=0, \quad (x + 9)(x - 8) = 0 \)
\( x = 8 \) is usable, \( y^2 = 64 - 1 = 63, \quad y = \sqrt{63} = 3\sqrt{7} \),
perimeter = \( 12 + 12 + 2(3\sqrt{7}) = 24 + 6\sqrt{7} \)

24. The path makes 4 quarter circles of radius = 2 at the corners and contains four sides
of a square that has sides of 5. The length of the path = \( 2\pi(2) + 4(5) = 4\pi + 20 \)

25. \( 14^2 = 6^2 + (CH)^2, \quad 160 = (CH)^2, \quad CH = 4\sqrt{10} \)
by \( \square s, \quad \frac{DE}{DE + 4\sqrt{10}} = \frac{4}{10} \)
\( 4DE + 16\sqrt{10} = 10DE, \quad 6DE = 16\sqrt{10} \)
\( DE = \frac{16\sqrt{10}}{6} = \frac{8\sqrt{10}}{3} \)

26. Solution: \( 10 + 2x = 26 \quad \Rightarrow \quad 2x = 16 \quad \Rightarrow \quad x = 8 \)
The coordinates of C are \( (8, 24 + 18) \) or \( (8, 42) \)
The coordinates of A are \( (18, 18) \)

27. The center of the circle is \( (2,-3) \). The slope of the radius to \( (8, -8) \) is \( m = \frac{-8 + 3}{8 - 2} = \frac{-5}{6} \).
The slope of the tangent at \( (8, -8) = \frac{6}{5} \). The equation of the tangent is
\( (y + 8) = \frac{6}{5}(x - 8) \quad \Rightarrow \quad y = \frac{6}{5}x - \frac{88}{5} \)

28. The grazing area consists of \( \frac{3}{4} \) of a circle with radius 100 and
\( 2 \left( \frac{1}{4} \text{ of circle of radius 20} \right) \Rightarrow A = \frac{3}{4}(\pi)(10000) + \frac{1}{2}(\pi)(400) = 7700\pi \)
29. Solution: $x^2 + 12^2 = (x + 5)^2 \Rightarrow x^2 + 144 = x^2 + 10x + 25 \Rightarrow 10x = 119 \Rightarrow x = 11.9 \Rightarrow x + 5 = 16.9$

30. From every point of the $n$ points an arc can be named to $n-1$ points, but a major and minor arc can be named in each case.

So, there are $\frac{n(n-1)}{2} = n(n-1)$

Tiebreaker 1: $m\angle MTR = 21, \ m\overline{OL} = 42, \ m\angle NQT = \frac{1}{2}(138 - 42) = \frac{1}{2}(96) = 48. \ m\overline{FN} = 48, \ m\overline{MN} = 180 - 42 - 48 = 90$

Tiebreaker 2.

Using the common tangent procedure, by inscribed angles, $m\overline{PR} = 86^\circ \Rightarrow m\angle PAR = 86 \Rightarrow m\angle ZOA = 4^\circ \Rightarrow m\angle ROT = 90^\circ + 4^\circ = 94^\circ$

$m\angle S = \frac{1}{2}(94) = 47^\circ$

Tiebreaker 3. In the smaller circle, $70 = \frac{1}{2}(m\overline{BD} + 20) \Rightarrow \frac{1}{2}m\overline{BD} = 60 \Rightarrow m\overline{BD} = 120.$

In the larger circle, $70 = \frac{1}{2}(160 - m\overline{BD}) \Rightarrow 140 = 160 - m\overline{BD} \Rightarrow m\overline{BD} = 20$

$\therefore 120 - 20 = 100$