Note: Figures are not drawn to scale.

- 1. A secant segment which passes through the center of a circle and a tangent segment form an angle of measure 30 degrees. If the tangent segment has a measure of 6, what is the measure _of the secant segment?
- a) $2\sqrt{3}$ b) $4\sqrt{3}$ c) $6\sqrt{3}$ d) 8 e) NOTA
- 2. A chord 30 centimeters long is drawn perpendicular to the diameter of a circle. The radius of the circle has length 17 centimeters. Find the length of the shorter segment into which the chord divides the diameter.
- a) 5 b) 9 c) 15 d) 25 e) NOTA
- 3. Given the circle A as shown with $AB \perp AC$ and AB = 4, find the area of the segment.



- a) 4π b) $4\pi 4\sqrt{2}$ c) $4\pi 4$ d) $4\pi 8$ e) NOTA
- 4. The radii of 2 circles are 7 and 2. The distance between their centers is 13. Find the length of their common external tangent.
- a) $10\sqrt{3}$ b) $\frac{25}{2}$ c) 12 d) $\frac{23}{2}$ e) NOTA
- 5. If the center of the circle with equation $x^2 + y^2 + ax + by 20 = 0$ is the point (4,-8), find the length of the radius.
- a) $2\sqrt{5}$ b) $3\sqrt{5}$ c) $4\sqrt{5}$ d) 10 e) NOTA
- 6. Given the circle with arc lengths
 m BE = y, m BD = 6y, m DE = 8y. Find the measure of ∠C.
 a) 30 b) 40 c) 45 d) 50 e) NOTA



- The distance between the centers of two circles is 30. The circles have radii of lengths 8 and 12.
 Find the length of the common internal tangent with one endpoint on each circle.
- a) $4\sqrt{5}$ b) $6\sqrt{5}$ c) $8\sqrt{5}$ d) $10\sqrt{5}$ e) NOTA

8. Figure BCDEF is a regular polygon. *EG* is tangent to circle A at E. Find the sum of the measures of ∠CEG and ∠ECF.
a) 72 b) 108 c) 144 d) 162 e) NOTA



9. Circles A and C are externally tangent with radii of lengths 6 and 2 respectively. Let a common external tangent line intersect circle A at B and circle C at D. Find the area of figure ACDB.

a) $4\sqrt{3}$ b) $8\sqrt{3}$ c) $16\sqrt{3}$ d) $24\sqrt{3}$ e) NOTA

10. In the figure, BC = 6. FB = 10, DE = 12. Find FD. a) 17 b) 18 c) 19 d) 20 e) NOTA



A square is inscribed in a circle of radius 5. A second square is inscribed in a semi-circle of radius 5. Find the ratio of the first square's area to the second square's area.

a) 2:1 b) 4:1 c) 5:2 d) 5:4 e) NOTA

12. Given tangent circles A and C with tangents at points B, D, and E as shown. If the radius of circle A is half the radius of circle C and BF = 11, find FE.



a) 11.5 b) 12 c) 12.5 d) 13 e) NOTA

13. Given the figure with the lengths shown.

Find x.

a) $\frac{5}{4}$ b) $\frac{3}{2}$ c) $\frac{7}{4}$ d) 2 e) NOTA



14. In the figure $\overline{BE} \parallel \overline{CD}$, $m \overline{BC} = 120^{\circ}$, and $m \overline{EB} = 40^{\circ}$, then $m \angle EFB = ?$ a) 15° b) 20° c) 30° d) 40° e) NOTA

15. The area of a circle is 36π . The area of an equilateral triangle inscribed

in the circle is

a) $27\sqrt{3}$ b) $54\sqrt{3}$ c) $27\sqrt{2}$ d) $18\sqrt{2}$ e) NOTA

- 16. In the figure \overrightarrow{DB} and \overrightarrow{DC} are tangent to Circle A at points B and C respectively. If the $m \angle ABC = 40^\circ$, then $m \angle D$ equals
- a) 20° b) 40° c) 60° d) 80° e) NOTA
- 17. Given that m∠F = 30⁰. m CD = 150⁰, and chords CB and DE are equal in length. m∠ CGD = ?
 a) 45° b) 60° c) 90° d) 120° e) NOTA
- 18. Given that \overline{DB} and \overline{DC} are tangent to Circle A. $m \angle D = 56^{\circ}$, and $\overline{BE} \cong \overline{CE}$, Find $m \angle ABE$. a) 25° b) 28° c) 31° d) 34° e) NOTA

•

 In the given figure. Find the ratio of the area of the circle circumscribed about the square to the area of the circle inscribed in the square.

a)
$$\frac{3}{2}$$
 to 1 b) 2 to 1 c) $\frac{5}{2}$ to 1 d) π to 1 e) NOTA

20. Circles centered at A and D intersect at just the two points B and C, and A is on the circle centered at D. If AB = 1 and $\angle CAB = 60^\circ$, find the length AD.

a)
$$\frac{\sqrt{3}}{3}$$
 b) $\frac{\sqrt{3}}{2}$ c) $\sqrt{3}$ d) 2 e) NOTA









National MAO 2009



a) 35 b)
$$\sqrt{1190}$$
 c) 45 d) $\sqrt{2115}$ e) NOTA

- 22. In circle A, CD = 4, BC = 10, and DA = 15. Find DE
- a) 2 b) 3 c) 5 d) 13 e) NOTA
- 23. Given: $\triangle BCD$ is isosceles with $\overline{BC} \cong \overline{BD}$. Circle A, $\overline{AF} \perp \overline{BD}$, $\overline{AE} \perp \overline{CD}$, BE = 6, AE = 1. Find the perimeter of $\triangle BCD$. a) a) 24 + 2 $\sqrt{3}$ b) 24 + 3 $\sqrt{5}$ c) 24 + 5 $\sqrt{3}$ d) 24 + 6 $\sqrt{7}$ E) NOTA



24. A circle of radius 2 rolls completely around the outside of a square with side of length 5, and returns to its starting point. (The circle and the square are coplanar.) What is the length of the path made by the center of the circle?

a) $\pi + 10$ b) $4\pi + 10$ c) $\pi + 20$ d) $4\pi + 20$ e) NOTA

25. Given: Externally tangent Circles A and C with $\overline{\text{DF}}$ tangent at E and F, AG = 10, CE = 4. Find DE.

a) 14 b)
$$\frac{6\sqrt{10}}{5}$$
 c) $6\sqrt{10}$ d) $\frac{8\sqrt{10}}{3}$ e) NOTA



National MAO 2009

- 26. Circles C and A are tangent to each other and to the axes as shown. AC = 26, BD = 24. Find the coordinates of C and A.
- a) (10,34) (15,15) b) (8,42) (18,18) c) (8,34) (15,15)
- d) (10,42) (15,15) e) NOTA



27. For the circle with equation $(x-2)^2 + (y+3)^2 = 61$. Write in slope-intercept form, the equation of the tangent to the circle at point (8, -8).

a)
$$y = \frac{6}{5}x - \frac{88}{5}$$
 b) $y = \frac{5}{6}x - \frac{48}{5}$ c) $y = \frac{5}{6}x + \frac{48}{5}$ d) $y = -\frac{5}{6}x - \frac{48}{5}$ e) NOTA:

28. A goat is tethered by a 100 ft. rope attached to an outside corner of an 80 ft. by 80 ft. square barn. How much grazing area outside the barn can the goat reach?

a) 7300π b) 7500π c) 7600π d) 7700π e) NOTA

29. An isosceles triangle with each leg measuring 13 is inscribed in a circle.

If the altitude to the base of the triangle is 5, find the radius of the circle.

a) 15.5 b) 15.9 c) 16.5 d) 16.9 e) NOTA

 If the points are selected on a given circle, find a formula for the number of arcs formed—including major and minor arcs and semicircles.



a)
$$\frac{n(n-1)}{2}$$
 b) $n(n-1)$ c) $\frac{n^2}{2}$ d) $\frac{[n(n-1)]^2}{6}$ e) NOTA

Theta Circles

Tiebreaker 1.

Given that circles *P* and *Q* are internally tangent at *T*. $m\overline{MR} = 42^{\circ}$ Diameter \overline{NS} if Circle *Q* is tangent to circle *P* at *O*. \overline{TM} passes through *O*. Find $m\overline{MN}$.

Tiebreaker 2. Given that Circle A is tangent to Circle O at R. $\overline{\text{QS}}$ passes through R. $\overline{\text{PT}}$ is a common external tangent at P and T. $m \angle Q = 43^{\circ}$; Find $m \angle S$





Tiebreaker 3.

The two circles intersect at *B* and *D*. If $m \angle DXB = 70^{\circ}$, $m \overline{FG} = 20^{\circ}$, and *m* arc *EH* = 160[°], find the difference between the measures of \overline{BD} of the smaller circle and \overline{BD} of the larger circle.

