2009 Equations and Inequalities Theta Topic Test

1. According to Descartes' Rule of Signs, the greatest possible number of imaginary roots for $5x^5 - 4x^3 + 8x^2 + 7x - 110 = 0$ is:

   A. 0 B. 1 C. 2 D. 3 E. NOTA

2. The solution set of $4x + 7 < 2x + 50$ does not contain:

   A. 21 B. 21.4 C. 21.49 D. 21.5 E. NOTA

3. Which of the following is not contained between the roots of the equation $2x^2 - 8x + 1 = 0$?

   A. 0 B. 1 C. 2 D. 3 E. NOTA

4. Find the sum of the reciprocals of the roots to $5x^5 - 4x^3 + 8x^2 + 7x - 110 = 0$.

   A. $\frac{7}{110}$ B. $-\frac{7}{110}$ C. $\frac{5}{4}$ D. $-\frac{5}{4}$ E. NOTA

5. Find the product of the *x*-values which satisfy $\begin{vmatrix} 3 & x & 6 \\ -x & 2 & 0 \\ 1 & x & x^2 \end{vmatrix} = 7$.

   A. $\frac{5}{4}$ B. $-\frac{4}{5}$ C. 0 D. 5 E. NOTA

6. The sum of the solutions to $27^{5x+3} = 81^{|x+2|-7}$ is closest to:

   A. -7 B. -5 C. -3 D. -2 E. NOTA

7. Find the sum of the solutions to $27^{|x+2|^2} = 9^{|x+2|+2}$.

   A. -8 B. -4 C. $-\frac{11}{3}$ D. $-\frac{22}{3}$ E. NOTA

8. If $xy = x - y$, then *y* cannot be:

   A. -1 B. 2 C. real D. 0 E. NOTA

9. Which of the following statements best describes $4x^2 - 3x - 1 = 0$?

   A. Its roots are imaginary. B. Its roots are real. C. Its roots are rational.
   D. Its roots are irrational. E. NOTA

10. Find x if $\cfrac{5}{3+\cfrac{5}{3+\cfrac{5}{3+...}}} = x$.

A. $\frac{-3+\sqrt{29}}{2}$ B. $\frac{3+\sqrt{29}}{2}$ C. $\frac{-1+\sqrt{5}}{2}$ D. $\frac{1+\sqrt{5}}{2}$ E. NOTA

11. Find the solution set for $\frac{1}{|3x+1|} \geq 5$.

A. $\left(-\infty, -\frac{2}{5}\right) \cup \left(-\frac{4}{15}, \infty\right)$ B. $\left[-\frac{2}{5}, -\frac{4}{15}\right]$ C. $\left(-\infty, -\frac{2}{5}\right] \cup \left[-\frac{4}{15}, \infty\right)$
D. $\left[-\frac{2}{15}, -\frac{4}{15}\right]$ E. NOTA

12. The positive number which satisfies $\frac{\frac{1}{4}}{\frac{1}{x}} = \frac{1}{x}$ can be described by:

A. odd, prime B. odd, composite C. even, prime
D. even, composite E. NOTA

13. The solution set for $|2x+1| > |x-5|$ can be written in the form $(-\infty, A) \cup (B, \infty)$. Find the sum of the integer closest to A that is also in the solution set and the integer closest to B that is also in the solution set.

A. –5 B. –2 C. 2 D. 5 E. NOTA

14. The solution(s) to $4x^2+8x-2\sqrt{4x^2+8x-3} = 6$ are/is in the interval:

A. $(-10, -5)$ B. $[-6, 0]$ C. $[0, 6]$ D. $(5, 10)$ E. NOTA

15. The sum of the integers in the solution set of $|x^2-5x| < 6$ is:

A. 10 B. 15 C. 20 D. no sum E. NOTA

16. Find $\log_z w$ if $\begin{cases} \log_x w = 36 \\ \log_y w = 18 \\ \log_{xyz} w^2 = 12 \end{cases}$.

A. 6 B. 12 C. 3 D. 18 E. NOTA

17. The solution set to $\frac{\log(4x-156)}{12x+31} \le 0$ can be written in the form $(A, B]$. Find $B-A$.

A. 3 B. $\frac{110}{3}$ C. $\frac{251}{6}$ D. $\frac{1}{4}$ E. NOTA

18. The length *L* of a tangent drawn from a point *A* to a circle is 4/3 of the radius *r*. The shortest distance from *A* to the circle is:

A. $\frac{r}{2}$ B. r C. $\frac{L}{2}$ D. $\frac{2}{3}L$ E. NOTA

19. Triangle *ABC* has *D* as the midpoint of $\overline{AB}$, *E* a point on $\overline{BC}$, *F* a point on $\overline{AC}$, and *G* the intersection of $\overline{DE}$ and $\overline{BF}$. If $BE:EC = 2:3$ and $DG:GE = 5:8$, find $BG:GF$.

A. 2:3 B. 3:4 C. 4:5 D. 5:6 E. NOTA

20. Find *x* if $3^x - 3^{x-3} = 78\sqrt{3}$.

A. $3\sqrt{3}$ B. $81\sqrt{3}$ C. $\frac{9}{4}$ D. $\frac{3}{2}\sqrt{3}$ E. NOTA

21. If *p* and *q* are positive integers such that $\frac{7}{10} < \frac{p}{q} < \frac{11}{15}$, find the smallest possible value of *q*.

A. 13 B. 60 C. 30 D. 7 E. NOTA

22. If *x* and *y* are positive integers and $x + y + xy = 54$, find $x + y$.

A. 12 B. 14 C. 15 D. 16 E. NOTA

23. $\big||x-2|-1\big| = a$ has exactly three distinct roots. If *a* is an integer, then *a* must be:

A. 0 B. 1 C. 2 D. 3 E. NOTA

24. An elephant eats as much in one day as a rabbit does in one year (365 days). In one day, they together eat 111 kg of carrots. How many kilograms of carrots does the rabbit eat in one day?

A. $\frac{1}{2}$ B. $\frac{111}{165}$ C. $\frac{37}{122}$ D. $\frac{19}{61}$ E. NOTA

25. Hope and Gwen flew from Birmingham to Knoxville on Amnesia Airlines. Each lady forgot to pack lightly, and as a result they each paid an extra \$60 and \$100, respectively, for having luggage weighing over the "free" weight limit. Together, their luggage weighed 52 kg (about 115 pounds). If Hope had flown alone and taken the combined luggage, she would have paid an extra \$340. Find the maximum number of kilograms of luggage that a person flying on Amnesia Airlines can take (check in) without having to pay the extra charge.

A. 18 B. 20 C. 30 D. 15 E. NOTA

26. The solution interval for $x^4 - 9x^2 + 4x + 12 < 0$ can be written in the form (*A*, *B*). Find $A + B$.

A. 0 B. no sum C. –2 D. –4 E. NOTA

27. Bonnie the Builder needs 10,000 bricks to finish a job. From past experience she knows that at most 7% of a load of bricks will arrive damaged. If one load contains 100 bricks, what is the minimum number of bricks she should order to insure that she has enough to finish the job? (Bricks must be ordered by full loads.)

A. 10800 B. 10700 C. 10600 D. 10500 E. NOTA

28. Two runners, moving in the same direction around a circular track, each at a constant speed, pass every 12 minutes. If the faster of the two completes a lap in 10 seconds less than the other, what fraction of the track does the faster runner complete in one second?

A. $\frac{1}{80}$ B. $\frac{1}{72}$ C. $\frac{1}{70}$ D. $\frac{1}{36}$ E. NOTA

29. Find *abc* if $a + b + c = 0$ and $a^3 + b^3 + c^3 = 216$.

A. 48 B. 72 C. 24 D. 216 E. NOTA

30. The system of equations $\begin{cases} x^2 + xy + 3y^2 = 15 \\ -5x^2 + 31xy - 3y^2 = 45 \end{cases}$ has two ordered-pair solutions (*x*, *y*) that, if graphed, would be located in Quadrant I. Find the distance between these two points.

A. $2\sqrt{7}$ B. $\sqrt{13}$ C. $\sqrt{5}$ D. $\sqrt{37}$ E. NOTA

TB1. Find the circumference of the circle defined by $x^2 + y^2 = 6241$.

TB2. Name the specific polygon formed by the intersections of $y = 3x + 2$, $y = -3x + 2$ and $y = -2$.

TB3. For a certain group of people, the equation $N = 8 \cdot 10^8 \cdot x^{-\frac{3}{2}}$ gives the number of individuals whose income exceeds *x* dollars. The lowest income, in dollars, of the wealthiest 800 individuals is at least 10^P. Find *P*.