

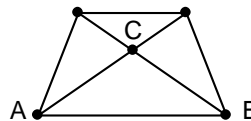
2009 Geometry Test (Theta)

1. How many lines of symmetry does a regular polygon of s sides have?
 A. $s - 3$ B. $s - 2$ C. $s - 1$ D. s E. NOTA

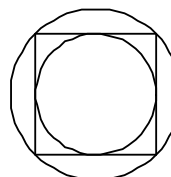
2. A band of uniform width is cut from three sides of a square. That leaves a rectangle whose area is one-half the area of the original square. What is the ratio of the width of the band to a side of the square?
 A. 1:2 B. 1:4 C. 1:8 D. 1:16 E. NOTA

3. What is the length, in inches, of a side of a regular hexagon inscribed in a circle whose diameter is 18 inches?
 A. 18 B. 9 C. $\frac{9\sqrt{3}}{2}$ D. 24 E. NOTA

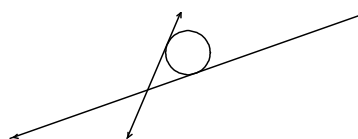
4. A trapezoid is formed by truncating an isosceles triangle. The area of the triangle is 60 square units. The area of the trapezoid is 45 square units. What is the area of triangle ABC?
 A. 18 B. 20 C. 22 D. 24 E. NOTA



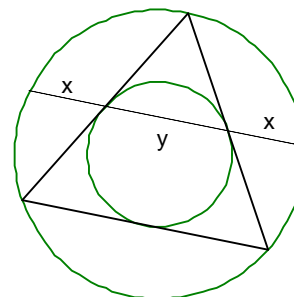
5. A square is inscribed in a circle and circumscribed about a smaller, concentric circle. What is the ratio of area of the inner circle to the area of the annulus (ring outside the inner circle and inside the outside circle)?
 A. 3:1 B. 3:2 C. 5:2 D. 7:1
 E. NOTA



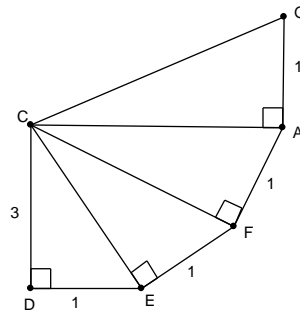
6. How many non-overlapping planar regions are formed when two lines containing a point exterior to circle are drawn tangent to the circle?
 A. 6 B. 5 C. 4 D. 3
 E. NOTA



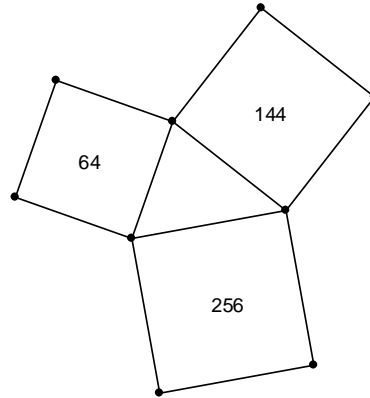
7. What is the ratio of x to y in the adjacent sketch if the solid chords of the larger circle are tangents to the smaller circle and the dotted chord contains two of the points of tangency on the smaller circle? The triangle is equilateral and the circles have the same center.
 A. $\frac{1+\sqrt{3}}{3}$ B. $\frac{1+\sqrt{5}}{2}$ C. $\frac{\sqrt{5}-1}{2}$ D. $\frac{3\sqrt{3}-3}{2}$
 E. NOTA



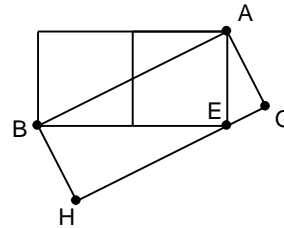
8. What is the length of \overline{CG} in the adjacent sketch?
 A. 7 B. $\sqrt{5}$ C. $\sqrt{13}$ D. $\sqrt{15}$
 E. NOTA



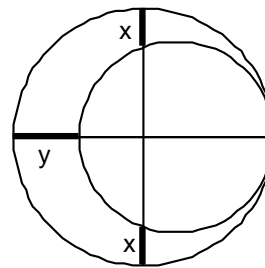
9. Find the area of the triangle in the adjacent figure. Each side of the triangle has a square constructed on it and with the area of the square as indicated.
 A. 64 B. 48 C. $16\sqrt{3}$ D. $12\sqrt{15}$
 E. NOTA



10. The two squares in the adjacent diagram have side length 2. $ABHG$ is a rectangle. Point E is a vertex of one of the squares and lies on \overline{GH} . Find the area of triangle BHE .
 A. $\frac{16}{5}$ B. $2\sqrt{5}$ C. 8 D. $\frac{4\sqrt{5}}{5}$
 E. NOTA



11. Two circles are drawn internally tangent, as shown. Two perpendicular diameters of the larger circle are drawn and the parts external to the smaller circle are x , of length 10, and y , of length 16. Find the area of the smaller circle.
 A. 625π B. 289π C. 169π D. 64π
 E. NOTA



12. If a square is inscribed in a semicircle, what is the ratio of a side of the square to the diameter of the semicircle?
 A. $\frac{\sqrt{5}}{1}$ B. $\frac{2\sqrt{5}}{5}$ C. $\frac{\sqrt{5}}{5}$ D. $\frac{\sqrt{5}}{10}$ E. NOTA

13. Three tennis balls are tightly packed in a cylindrical can, where the balls are stacked on top

of one another. What fraction of the volume of the can is unoccupied?

- A. $\frac{1}{2}$ B. $\frac{2}{9}$ C. $\frac{1}{3}$ D. $\frac{1}{4}$ E. NOTA

14. The perimeter of an isosceles triangle is 36 and the altitude to the base is 12. What is the area of the triangle?

- A. 60 B. 120 C. 13 D. 256 E. NOTA

15. A cube has a space diagonal whose measure in inches is the same as the volume of the cube in cubic inches. What is the length of an edge of the cube?

- A. $\sqrt{3}$ B. $\sqrt{2}$ C. $\sqrt[3]{3}$ D. $\sqrt[3]{2}$ E. NOTA

16. A square of perimeter 40 is inscribed in a square of perimeter 56. What is the greatest distance between a vertex of the inner square and a vertex of the outer square?

- A. 4 B. 6 C. 8 D. 10 E. NOTA

17. Each sheet of toilet tissue is 4 inches long, the diameter of the full roll is 6 inches and the diameter of the cardboard core is 2 inches. How many times (round a partial time to the next integer) does the toilet tissue wind around its cardboard core on a 300-sheet roll of bathroom tissue? Note: $\pi \approx \frac{22}{7}$.

- A. 325 B. 215 C. 127 D. 96 E. NOTA

18. Which of the following points is *not* always on the Euler line?

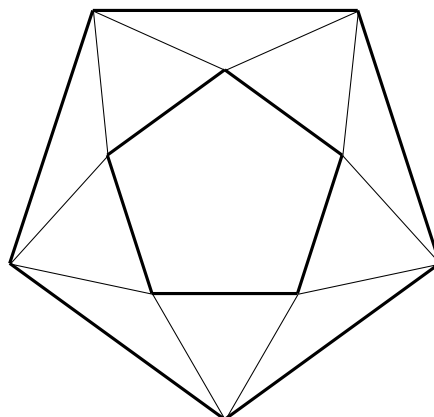
- A. centroid B. circumcenter C. incenter
D. center of nine-point circle E. NOTA

19. Which of the following is *not* one of the ancient impossible constructions in Euclidean geometry?

- A. squaring the circle B. doubling the square C. doubling the cube
D. trisecting an angle E. NOTA

20. Five equilateral triangles are constructed on the sides of a regular pentagon, exterior to the pentagon. When the outer vertices of these triangles are connected, a larger regular pentagon results, as shown. If the length of a side of the small pentagon is 1, then the length of a side of the large pentagon is $2\cos x$. What is the degree measure of the positive acute angle x ?

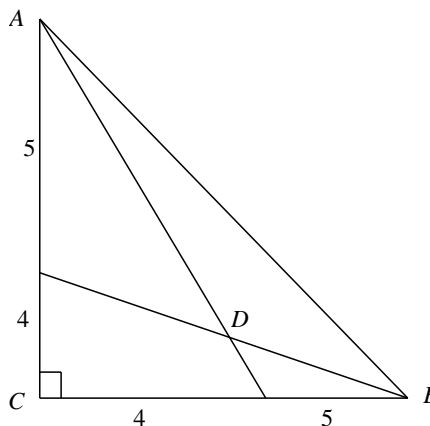
- A. 18 B. 30 C. 60
D. 66 E. NOTA



21. If a sector of a circle has perimeter 280 mm and an area of 49 cm^2 , what is the length, in centimeters, of the arc of this sector?
 A. 7 B. 14 C. $140 - 42\sqrt{11}$ D. not enough information E. NOTA
22. From a point within a triangle, line segments are drawn to the vertices. A necessary and sufficient condition that the three triangles thus formed have equal areas is that the point be:
 A. the center of the inscribed circle
 B. the center of the circumscribed circle
 C. such that the three angles formed at the point each measure 120°
 D. the intersection of the medians of the triangle
 E. NOTA
23. On a map whose scale is 400 miles to an inch and a half, a certain plot of land is represented by a rhombus having a 60° angle. The diagonal opposite 60° is $\frac{3}{16}$ in. The area of the plot of land in square miles is:
 A. $\frac{2500\sqrt{3}}{3}$ B. $\frac{1250\sqrt{3}}{3}$ C. 1250 D. $\frac{5625\sqrt{3}}{2}$ E. NOTA
24. The parallel sides of a trapezoid measure 3 and 9 units. The non-parallel sides measure 4 and 6 units. A line parallel to the bases divides the trapezoid into two trapezoids of equal perimeters. Find the ratio into which the two non-parallel sides is divided. (A 5:7 ratio, for example, could be considered 7:5.)
 A. 4:3 B. 4:1 C. 3:2 D. 3:1 E. NOTA

25. Find the area of $\triangle ABD$.

- A. not enough information B. 15
 C. $\frac{245}{16}$ D. $\frac{405}{26}$ E. NOTA



26. The maximum number of pieces a (cylindrical) cake can be cut into with five straight, planar cuts is:
 A. 13 B. 14 C. 15 D. 26 E. NOTA
27. Find the sine value of the dihedral angle between two faces of a regular tetrahedron.

- A. $\frac{\sqrt{3}}{2}$ B. $\frac{2}{3}\sqrt{3}$ C. $\frac{2}{3}\sqrt{2}$ D. $\frac{2}{3}\sqrt{6}$ E. NOTA

28. If the sides of a triangle are in the ratio 4:6:8, what is the cosine of the smallest angle?

- A. $\frac{7}{8}$ B. $\frac{3}{4}$ C. $\frac{11}{16}$ D. $-\frac{1}{4}$ E. NOTA

29. Two chords meet at point P on a circle. They intercept a 120° arc. If the chords measure 10 and 8, find the radius of the circle.

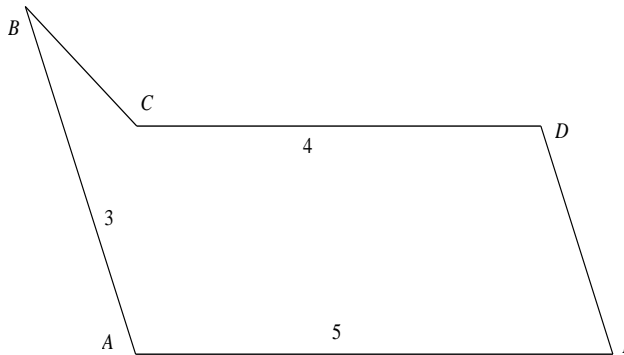
- A. $6 + \frac{2}{3}\sqrt{21}$ B. $2\sqrt{7}$ C. $4\sqrt{7}$ D. $8\sqrt{7}$ E. NOTA

30. Triangle ABC has point D on \overline{AC} and point E on \overline{BC} . $AD:AC = 1:4$. Point G is the intersection of \overline{AE} and \overline{BD} , and $BG:GD = 8:3$. If AG and AE are in the ratio $x:y$, find $x+y$.

- A. 11 B. 18 C. 17 D. 16 E. NOTA

T1. What is the first (positive) triangular number?

T2. Find the area of pentagon $ABCDE$ if $\overline{AE} \perp \overline{AB}$, $\overline{CD} \parallel \overline{AE}$, $\overline{BC} \parallel \overline{DE}$, $CD = 4$, $AB = 3$, $AE = 5$, and the distance from \overline{AE} to \overline{CD} is 2.



T3. What is the area of a circle in which the lengths of consecutive sides of an inscribed hexagon are 1, 1, 1, 2, 2, and 2?