2009 Logs and Exponents Topic Test (Theta)

Solutions:

1. $\frac{\log x}{\log 2} + \frac{\log x}{2\log 2} + \frac{\log x}{4\log 2} = 7; \frac{4\log x + 2\log x + \log x}{4\log 2} = 7; \frac{\log x^7}{4\log 2} = 7 \rightarrow \log x = \log 16; x = 16$ ***D***

2. $log\left(\frac{27}{25}\right) = log\ 27 - log\ 25 = 3\ log\ 3 + 2\ log\ 5, 3(1.5) - 2(2.2) = 0.1$ ***E***

3. $\frac{128^{(x-1)}}{4^{(2x+2)}} = 32^x; 2^{(7x-7)} \cdot 2^{-(4x+4)} = 2^{5x}; 2^{(3x-11)} = 2^{5x} \rightarrow 3x - 11 = 5x; x = -11/2. \frac{x}{2} = -11/4$ ***C***

4. $\frac{2^{2^3}}{2^{2^2}} = \frac{2^8}{2^4} = 4; \frac{3^{3^3}}{3^{3^2}} = \frac{3^{27}}{3^9} = 27; 2^4 + 3^{16} = 16 + 3^{16};$ ***E***

5. $log_{12} x^2 = log_{12} 3 + log_{12} 27; x^2 = 81; x = \pm 9$. Defined for only 9. ***B***

6. $log\ x^{\frac{1}{2}} - \log\frac{3}{2x} + log\ x = \frac{1}{2}\ log\ x - log\ 3 + log\ 2x + log\ x; 3/2 log\ x - log\ 3 + log\ 2 + log\ x;$
   $\frac{5}{2} log\ x + log\ \frac{2}{3} \rightarrow \frac{23}{6}$ ***B***

7. $log\frac{x+4}{2x-3} = log\ 2; \frac{x+4}{2x-3} = 2; x + 4 = 4x - 6 \rightarrow x = \frac{10}{3}$. ***C***

8. $(6^2 - 10^2)^{-\frac{1}{2}} = (36 - 100)^{-\frac{1}{2}}$; Undefined ***E***

9. $x = 3\cdot 2^{(y+1)}; \frac{x}{3} = 2^{(y+1)}; log_2\frac{x}{3} = y + 1 \rightarrow log_2\frac{x}{3} - 1 = f^{-1};$ ***A***

10. $log_4(2) = log_5 x; \frac{1}{2} = log_5 x \rightarrow x = \sqrt{5}$. ***A***

11. $2^{3x+1} \cdot 3^{5y+7} = \frac{16}{27}; 3x + 1 = 4; x = 1; 5y + 7 = -3; y = -2; 2 + 6 = 8$ ***C***

12. $\frac{1}{2}ln\ 9 + ln\ 12 - 2ln\ 3 = ln\ 3 + 2ln\ 2 + ln\ 3 - 2ln\ 3; ln\ 4$ ***B***

13. $\left(10^{4n^2+8} + 1\right)^2 \rightarrow$ Let $p = 4n^2 + 8$, then 10^p = a power of ten and only 1 and following zeros possible. Let $n = \frac{1}{2}$ then $p = 9$ and $(10^9 + 1)^2 \rightarrow (1000..1)^2$ and sum of digits $2^2 = 4$. ***D***

14. $2^a = 2^{-2}; 2^{-b} = 2^5; 2^{1/2c} = 2^2; a = -2, b = -5, c = 4 \rightarrow -2 - 5 - 4 = -11$ ***B***

15. $\log 80 = a$; $\log(8\cdot 10) = a$; $a = 1 + 3\log 2$; ;$b = \log\left(\frac{90}{2}\right)$;$b = 1 + \log 9 - \log 2$
$a + b - 2 = 2\log 2 + 2\log 3 = \log 36$. ***D***

16. By inspection: ***C***,

17. $6 + {}^{7}/_{3} + 2 - 1 = {}^{28}/_{3}$. Ave: ${}^{7}/_{3}$. ***E***

18. $-2 < \log_2 x < 16$; $2^{-2} < x < 2^{16}$; They are all in the solution set. ***E***

19. $\log_3(3x+4) = \log_3(x-2) + 2$; $\log_3\left(\frac{3x+4)}{x-2}\right) = 2$; $\left(\frac{3x+4)}{x-2}\right) = 9$; $x = {}^{11}/_{3}$, $y = \log_3 15$.
$({}^{11}/_{3}, \log_3 15)$. ***A***

20. $\ln\left(\sqrt{\frac{x^2-1}{x+1}}\right) = 2$. $e^4 = \frac{(x-1)(x+1)}{x+1}$; $\rightarrow x = e^4 + 1$. ***C***

21. $\log_5 x + \log_7 x = \frac{\log x}{\log 7} + \frac{\log x}{\log 7} \rightarrow \frac{\log x\,(\log 7 + \log 5)}{(\log 7)(\log 5)}$, III only. ***E***

22. $\log\frac{1}{2} + \log\frac{2}{3} + \log\frac{3}{4} + \ldots + \log\frac{998}{999} + \log\frac{999}{1000}$; $\log\left(\frac{1}{2}\cdot\frac{2}{3}\cdot\frac{3}{4}\ldots\frac{998}{999}\cdot\frac{999}{1000}\right)$, $\log\left({}^{1}/_{1000}\right) = -3$ ***C***

23. $(\log 4)^3 + 3(\log 4)^2(\log 25) + 3(\log 4)(\log 25)^2 + (\log 25)^3 = (\log 4 + \log 25)^3 \rightarrow (\log 100)^3 = 2^3$;
8. ***A***

24. $\log(\log x) = 3$; $10^3 = \log x$; $10^{1000} = x$, 1000 zeros + 1 = 1001; ***C***

25. $x = \frac{\log 9}{\log 4}\cdot\frac{\log 12}{\log 9}\cdot\frac{\log 32}{\log 12}$; $x = \frac{5\log 2}{2\log 2} = {}^{5}/_{2}$. ***C***

26. $5^{x^2-9x} = 5^{-18}$; $x^2 - 9x + 18 = 0$; $x = 3, 6$. ***D***

27. $(\log_4 x)^2 + \log_4 x = 6$; $(\log_4 x + 3)(\log_4 x - 2) = 0$. $x = 4^{-3}$, 16. ***B***

28. $4^{3m-1} = 1 + 8^{2(k+3)}$; $2^{6m-2} - 2^{6k+18} = 1 \rightarrow 6m - 2 = 2$, $m = \frac{1}{2}$; $6k + 18 = 0$, $k = -3 \rightarrow m^k = \left(\frac{1}{2}^{-3}\right)$
$m^k = \left(\frac{1}{8}\right)$; ***E***

29. $P(t) = \frac{800}{1+49e^{-0.2t}}$; P(t)($1 + 49e^{-0.2t}$) = 800, $P(t) + P(t) \cdot 49e^{-.2t} = 800$,

$49e^{-.2t} = 800 - P(t)$, $e^{-.2t} = \frac{800 - P(t)}{P(t) \cdot 49}$ $\rightarrow -.2t = ln\left(\frac{800-P(t)}{P(t)49}\right) \rightarrow t = -5\, ln\left(\frac{800-P(t)}{P(t)49}\right)$. ***E***

30. $log_7 \frac{(x^3+27)}{(x+3)} = 2$; $\frac{(x^3+27)}{(x+3)} = 49$; $x^3 - 49x - 120 = (x-8)(x^2+8x+15)$, $x = 8$. ***D***

Tie-Breakers:

1. $f(3) = log_2(9) - log_2(9) - 5$; **-5**

2. $log_a x + log_{a^2} x + log_{a^4} x \rightarrow log_a x + log_{a^2} a \cdot log_a x + log_{a^4} a \cdot log_a x = c$; $log_a x\left(1 + \frac{1}{2} + \frac{1}{4}\right) = c$

$log_a x\left(\frac{7}{4}\right) = c$; $log_a x = \left(\frac{4}{7}\right)c$; $x = \boldsymbol{a^{\frac{4}{7}c}}$

3. $2^{3x+1} = 16^{2x-1}$; $3x + 1 = 8x - 4$; $x = 1$; $3^{2y-1} = 27^{4y+3}$; $2y - 1 = 12y + 9$; $y = -1$; **(1, -1)**