Solutions:

1. $B^T = \begin{bmatrix} 1 & 0 & -3 \\ -2 & 3 & 1 \end{bmatrix}$, $AB^T = \begin{bmatrix} 0 & -1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 1 & 0 & -3 \\ -2 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -3 & -1 \\ -3 & 6 & -1 \end{bmatrix}$. **C**

2. $\begin{bmatrix} 1 & a \\ 0 & b \end{bmatrix}\begin{bmatrix} 1 & 0 \\ a & b \end{bmatrix} = \begin{bmatrix} 1+a^2 & ab \\ ab & b^2 \end{bmatrix}$. Since the resulting matrix is the identity matrix, ab=0 and $1+a^2 = b^2 = 1$.
a = 0 and $b = \pm 1$. Take the absolute value and get (0, 1). **D**

3. $\begin{vmatrix} 1 & 2 & -3 & 0 \\ -2 & 1 & 0 & 3 \\ 0 & 4 & -4 & -1 \\ 0 & -1 & 2 & 5 \end{vmatrix} = \begin{vmatrix} 1 & 2 & -3 & 0 \\ -2 & 1 & 0 & 3 \\ 0 & 4 & -4 & -1 \\ 0 & -1 & 2 & 5 \end{vmatrix} = \begin{vmatrix} 1 & 2 & -3 & 0 \\ 0 & 5 & -6 & 3 \\ 0 & 4 & -4 & -1 \\ 0 & -1 & 2 & 5 \end{vmatrix} = \begin{vmatrix} 0 & 4 & 28 \\ 0 & 4 & 19 \\ -1 & 2 & 5 \end{vmatrix} = \begin{vmatrix} 0 & 0 & 9 \\ 0 & 4 & 19 \\ -1 & 2 & 5 \end{vmatrix}$ = -1(-36) = 36

C

4. x has roots 2 and -4, y is -7. Using the positive root of x, xy = (-7)(2) = -14. **E**

5. Evaluate $\frac{1}{2}\begin{vmatrix} 2 & -3 & 1 \\ 3 & -5 & 1 \\ 1 & 4 & 1 \end{vmatrix} \to \frac{1}{2}\left|(-10-3+12)-(-5+8-9)\right| \to \frac{5}{2}$ **B**

6. $\begin{vmatrix} x & 0 & c \\ -1 & x & b \\ 0 & -1 & a \end{vmatrix} = 0 \to ax^2 + bx + c.$ Sum of roots of x is $\frac{-b}{a}$, and product of roots of x is $\frac{c}{a}$. $\frac{c}{a} - \left(-\frac{b}{a}\right) = \frac{c+b}{a}$.
C

7. $\begin{vmatrix} -5 & 9 \\ 7 & 6 \end{vmatrix}$ = (-30 – 63) = -93 **B**

8. Det $\begin{bmatrix} 4 & 5 & -1 \\ -3 & 2 & 0 \\ 3 & -2 & 1 \end{bmatrix}$ = 4(2) – 5(-3) – 1(6 – 6) = 23 → b = 23. $A^{-1} = \frac{1}{23}\begin{bmatrix} 2 & -3 & 2 \\ 3 & 7 & 3 \\ 0 & 23 & 23 \end{bmatrix}$ **E**

9. Using elementary row operations $\begin{bmatrix} 1 & 1 & 3 \\ 2 & 0 & 1 \\ 1 & -1 & -2 \end{bmatrix} \to \begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 3 \\ 1 & -1 & -2 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 1 & 1 & 3 \\ 1 & -1 & -2 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & -1 & -\frac{5}{2} \end{bmatrix} \to \begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & 0 & 0 \end{bmatrix}$ **C**

10. $\begin{bmatrix}-2 & -3\\ 1 & 2\end{bmatrix}^{-1} = -1\begin{bmatrix}2 & 3\\ -1 & -2\end{bmatrix}$; $\begin{bmatrix}-2 & -3\\ 1 & 2\end{bmatrix}\begin{bmatrix}5 & 3\\ -2 & 1\end{bmatrix} = \begin{bmatrix}-4 & -9\\ 1 & 5\end{bmatrix}$ **E**

11. Singular matrices have determinants of 0. $\begin{vmatrix}1 & 4 & 2\\ 3 & 3 & 2\\ -1 & y & 0\end{vmatrix} = 0 \rightarrow (-8+6y)-(-6+2y)=0$, $y=\frac{1}{2}$ **C**

12. $\begin{bmatrix}\frac{1}{2} & -\frac{\sqrt{3}}{2}\\ \frac{\sqrt{3}}{2} & \frac{1}{2}\end{bmatrix}\begin{bmatrix}0\\ 1\end{bmatrix} = \begin{bmatrix}-\frac{\sqrt{3}}{2}\\ \frac{1}{2}\end{bmatrix}$. 60° counterclockwise. **E**

13. $det\begin{bmatrix}2i & -i & 3\\ i & 4 & -2\\ -3 & i & 4\end{bmatrix} = 2i(16+2i)+i(4i-6)+3(-1+12) = 32i-4-4-6i-3+36 = 25+26i.$

$2a - 3b = 50 - 78 = -28.$ ***D***

14. To solve for *z*, the coefficients of *z* in the third column are replaced. **E**

15. Singular matrices are matrices with determinant of 0. I, II, V. **E**

16. $\begin{bmatrix}1-x & 3\\ 3 & 1-x\end{bmatrix} \rightarrow (1-x)(1-x)-9=0 \rightarrow x=4,-2.$ **C**

17. The resulting matrix is a 3x2 matrix; 6 elements. **A**

18. $\begin{vmatrix}1 & -1 & 2\\ 3 & -1 & 2\\ 1 & 2 & -K\end{vmatrix} = 0 \rightarrow (K+12-2)-(-2+4+3K)=0 \rightarrow K=4$ **A**

19. $\frac{4\cdot 2+3\cdot 1-2\cdot 1+3}{\sqrt{2^2+1^2+(-2)^2}} = \frac{12}{\sqrt{9}} = 4.$ **D**

20. $\left(\begin{bmatrix}1 & 4\\ 2 & 3\\ 3 & 2\end{bmatrix}\bullet\begin{bmatrix}1 & 0 & 2\\ 2 & 3 & 1\end{bmatrix}\right)^T = \begin{bmatrix}9 & 12 & 6\\ 8 & 9 & 7\\ 7 & 6 & 8\end{bmatrix}^T = \begin{bmatrix}9 & 8 & 7\\ 12 & 9 & 6\\ 6 & 7 & 8\end{bmatrix}$. **C**

21. **A**

22. **C**

23. **D**

24. $\begin{bmatrix}1 & -1\\1 & 1\end{bmatrix}^{11} \to \begin{bmatrix}1 & -1\\1 & 1\end{bmatrix}\left(\begin{bmatrix}1 & -1\\1 & 1\end{bmatrix}^2\right)^5 \to \begin{bmatrix}1 & -1\\1 & 1\end{bmatrix}\begin{bmatrix}0 & -2\\2 & 0\end{bmatrix}\left(\begin{bmatrix}0 & -2\\2 & 0\end{bmatrix}^2\right)^2 \to \begin{bmatrix}1 & -1\\1 & 1\end{bmatrix}\begin{bmatrix}0 & -2\\2 & 0\end{bmatrix}\begin{bmatrix}-4 & 0\\0 & -4\end{bmatrix}^2$

$\begin{bmatrix}1 & -1\\1 & 1\end{bmatrix}\begin{bmatrix}0 & -2\\2 & 0\end{bmatrix}\begin{bmatrix}-4 & 0\\0 & -4\end{bmatrix}^2 \to \begin{bmatrix}-32 & -32\\32 & -32\end{bmatrix}$. **C**

25. $3 \cdot 3^3 = 81$ **C**

26.

$$\begin{bmatrix}1&2&3\\0&4&5\\1&0&6\end{bmatrix}\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix} = \begin{bmatrix}1&2&3\\0&4&5\\0&-2&3\end{bmatrix}\begin{bmatrix}1&0&0\\0&1&0\\-1&0&1\end{bmatrix} = \begin{bmatrix}1&2&3\\0&4&5\\0&0&11\end{bmatrix}\begin{bmatrix}1&0&0\\0&1&0\\-2&1&2\end{bmatrix} = \begin{bmatrix}1&2&3\\0&4&5\\0&0&1\end{bmatrix}\begin{bmatrix}1&0&0\\0&1&0\\-2/11&1/11&2/11\end{bmatrix}$$

$$= \begin{bmatrix}1&2&3\\0&4&0\\0&0&1\end{bmatrix}\begin{bmatrix}1&0&0\\10/11&-5/11&-10/11\\-2/11&1/11&2/11\end{bmatrix} = \begin{bmatrix}1&2&3\\0&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}1&0&0\\5/22&-5/44&-5/22\\-2/11&1/11&2/11\end{bmatrix};$$ **A**

27. A 2x2 and a 3x2 matrix cannot be multiplied in that order. **E**

28. $3a + 6 = -6 - 18$, $a = -10$. **B**

29. $[(2)(1)(1)+(-1)(-2)(1)+(-3)(2)(0)]-[(0)(1)(1)+(2)(2)(-2)+(-3)(-1)(1)] = 9.$ **C**

30. $\begin{bmatrix}i & \pi\\e & 1\end{bmatrix}^{-1} = \dfrac{1}{i - e\pi}\begin{bmatrix}1 & -\pi\\-e & i\end{bmatrix} = \dfrac{1}{e\pi - i}\begin{bmatrix}-1 & \pi\\e & -i\end{bmatrix}$. **D**

Tiebreakers:

1. $\begin{bmatrix}1&0&2\\0&0&3\\2&1&0\end{bmatrix}$; The sum of the entries gives the directed paths, so **9** directed paths

2. $\begin{bmatrix}3 & -2 & 5\end{bmatrix} \bullet \begin{bmatrix}4\\6\\10\end{bmatrix} = [\mathbf{50}]$

3. $\begin{bmatrix} -2 & 1 & 0 \\ 5 & -3 & 1 \end{bmatrix} \bullet \begin{bmatrix} a & 0 \\ 3 & b \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} -3 & 10 \\ 8 & -31 \end{bmatrix} = \begin{bmatrix} -2a+3 & b \\ 5a-7 & -3b-1 \end{bmatrix} = \begin{bmatrix} -3 & 10 \\ 8 & -31 \end{bmatrix}$; $a = 3$, $b = 10$.

$a + b =$ **13**.