

2009 Matrices and Determinants Theta Topic Test

For all items, answer E, NOTA means “none of the above is correct.” Unless otherwise stated, “i” is to be interpreted as  $\sqrt{-1}$ .

1. Let  $A = \begin{bmatrix} 0 & -1 \\ 1 & 2 \end{bmatrix}$  and  $B = \begin{bmatrix} 1 & -2 \\ 0 & 3 \\ -3 & 1 \end{bmatrix}$ , find  $AB^T$ .

A.  $\begin{bmatrix} -3 & 2 \\ 6 & -3 \\ -1 & -1 \end{bmatrix}$

B.  $\begin{bmatrix} -2 & 3 & 1 \\ -5 & 6 & 5 \end{bmatrix}$

C.  $\begin{bmatrix} 2 & -3 & -1 \\ -3 & 6 & -1 \end{bmatrix}$

D.  $\begin{bmatrix} -2 & -5 \\ 3 & 6 \\ 1 & 5 \end{bmatrix}$

E. NOTA

Given that an orthogonal matrix is a square matrix  $A$  for  $AA^T = A^T A = I$ . (Use for #2 only)

2. For what absolute values of (a, b) would matrix  $A$  be orthogonal?  $A = \begin{bmatrix} 1 & a \\ 0 & b \end{bmatrix}$ .

A. (1, 1)

B. (0, 0)

C. (1, 0)

D. (0, 1)

E. NOTA

3. Find the value of  $\begin{vmatrix} 1 & 2 & -3 & 0 \\ -2 & 1 & 0 & 3 \\ 0 & 4 & -4 & -1 \\ 0 & -1 & 2 & 5 \end{vmatrix}$ .

A. 16

B. 28

C. 36

D. 42

E. NOTA

4. Given  $\begin{bmatrix} 1 & 3 & x^2 + 2x \\ 8 & -5 & 27 \end{bmatrix} = \begin{bmatrix} 5^0 & \sqrt[3]{27} & 8 \\ 2 \cdot 4 & x + y & 3^3 \end{bmatrix}$ , for a positive value of  $x$ , what is  $xy$ ?

A. 4

B. -6

C. 8

D. -12

E. NOTA

5. Find the area of the triangle with the vertices at (2, -3); (3, -5); and (1, 4).

A. 5

B.  $\frac{5}{2}$

C. -5

D. 7

E. NOTA

6. For the determinant  $\begin{vmatrix} x & 0 & c \\ -1 & x & b \\ 0 & -1 & a \end{vmatrix} = 0$ , let  $P$  be the product of roots of  $x$  and  $S$  be the sum of roots of  $x$ ,

find  $P-S$ .

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- A.  $ab + c$       B.  $\frac{c-2b}{2a}$       C.  $\frac{c+b}{a}$       D.  $2cb - a$       E. NOTA

7. Given that matrix  $A = \begin{bmatrix} -5 & 9 & 3 \\ 4 & -2 & 0 \\ 7 & 6 & -1 \end{bmatrix}$ , find the value of the minor of  $a_{(2,3)}$

- A. 0      B. -93      C. -58      D. -4      E. NOTA

8. Given matrix  $A = \begin{bmatrix} 4 & 5 & -1 \\ -3 & 2 & 0 \\ 3 & -2 & 1 \end{bmatrix}$ , then  $A^{-1} = \frac{1}{b} \begin{bmatrix} 2 & -3 & 2 \\ 3 & 7 & 3 \\ 0 & 1 & 1 \end{bmatrix}$ . Find the value of  $b$ .

- A. 12      B. 23      C. 31      D. -35      E. NOTA

9. Reduce  $\begin{bmatrix} 1 & 1 & 3 \\ 2 & 0 & 1 \\ 1 & -1 & -2 \end{bmatrix}$  to row-echelon form.

- A.  $\begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & -1 & -\frac{5}{2} \end{bmatrix}$       B.  $\begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 2 & \frac{5}{2} \\ 0 & 0 & -\frac{5}{2} \end{bmatrix}$       C.  $\begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & 0 & 0 \end{bmatrix}$       D.  $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & \frac{5}{2} \\ 0 & 0 & -\frac{5}{2} \end{bmatrix}$       E. NOTA

10. Find  $X$  if  $\begin{bmatrix} -2 & -3 \\ 1 & 2 \end{bmatrix} X = \begin{bmatrix} 5 & 3 \\ -2 & 1 \end{bmatrix}$

- A.  $\begin{bmatrix} 5 & 3 \\ -2 & 1 \end{bmatrix}$       B.  $\begin{bmatrix} -3 & -9 \\ 1 & 4 \end{bmatrix}$       C.  $\begin{bmatrix} -4 & -9 \\ -1 & 5 \end{bmatrix}$       D.  $\begin{bmatrix} -2 & -3 \\ 1 & 2 \end{bmatrix}$       E. NOTA

11. Find the value of  $y$  which makes  $\begin{bmatrix} 1 & 4 & 2 \\ 3 & 3 & 2 \\ -1 & y & 0 \end{bmatrix}$  a singular matrix.

- A.  $\frac{3}{4}$       B. 2      C.  $\frac{1}{2}$       D. 3      E. NOTA

12. The matrix  $\begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$  can be used to rotate points in the plane about the origin through the angle of...

- A. 30° clockwise                      B. 30° counterclockwise                      E. NOTA  
 C. 45° clockwise                      D. 45° counterclockwise

13. The value of  $\det \begin{bmatrix} 2i & -i & 3 \\ i & 4 & -2 \\ -3 & i & 4 \end{bmatrix} = a + bi$ . Find  $2a - 3b$ .

- A. -2                      B. -8                      C. -22                      D. -28                      E. NOTA

14. For  $\begin{cases} 2x + y - 3 = 1 \\ x + 2y - z = 2 \\ -y - z = -3 \end{cases}$ , if the solution for the variable “z” is written using Cramer’s rule, the numerator would be:

- A.  $\begin{vmatrix} 1 & 1 & -3 \\ 2 & 2 & -1 \\ -3 & 0 & 1 \end{vmatrix}$                       B.  $\begin{vmatrix} 2 & 1 & -3 \\ 1 & 2 & -1 \\ 0 & 0 & 1 \end{vmatrix}$                       C.  $\begin{vmatrix} 2 & 1 & -3 \\ 1 & 2 & -1 \\ 0 & -3 & 1 \end{vmatrix}$                       D.  $\begin{vmatrix} 2 & 1 & 1 \\ 1 & 2 & 2 \\ 0 & -1 & -3 \end{vmatrix}$                       E. NOTA

15. Which of the following matrices is (are) singular? ( $x, y \neq 0$ )

- I.  $\begin{bmatrix} x & x \\ y & y \end{bmatrix}$                       II.  $\begin{bmatrix} -1 & 0 \\ 1 & 0 \end{bmatrix}$                       III.  $\begin{bmatrix} x & -y \\ x & y \end{bmatrix}$                       IV.  $\begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}$                       V.  $\begin{bmatrix} x & 1 \\ x & 1 \end{bmatrix}$

- A. III, IV                      B. II, IV                      C. I, V                      D. III                      E. NOTA

16. Find the eigenvalue(s) of  $A = \begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix}$ .

- A. 2                      B. -3, 1                      C. -2, 4                      D. 10                      E. NOTA

17. Matrix A has 2 rows, 4 columns. Matrix B has 4 rows, 3 columns. The existing product of these two matrices will consist of P elements. What is the value of P?

- A. 6                      B. 8                      C. 12                      D. 16                      E. NOTA

18. Find the smallest real value of  $K$  so that the system of equations below has no solution.

$$\begin{aligned}x - y + 2z &= 1 \\3x - y + 2z &= 0 \\x + 2y - Kz &= 0\end{aligned}$$

- A. 4                      B. -2                      C. -7                      D. 2                      E. NOTA

19. What is the distance between the point  $(4, 3, 1)$  and the plane  $2x + y - 2z + 3 = 0$ ?

- A. 6                      B. 8                      C.  $\frac{12}{\sqrt{26}}$                       D. 4                      E. NOTA

20. Evaluate:  $\left( \begin{bmatrix} 1 & 4 \\ 2 & 3 \\ 3 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 2 \\ 2 & 3 & 1 \end{bmatrix} \right)^T$

- A.  $\begin{bmatrix} 9 & 12 & 6 \\ 8 & 9 & 7 \\ 7 & 6 & 8 \end{bmatrix}$                       B.  $\begin{bmatrix} 9 & 8 & 7 \\ 12 & 9 & 7 \\ 6 & 6 & 8 \end{bmatrix}$                       C.  $\begin{bmatrix} 9 & 8 & 7 \\ 12 & 9 & 6 \\ 6 & 7 & 8 \end{bmatrix}$                       D.  $\begin{bmatrix} 1 & 0 & 2 \\ 1 & 2 & 3 \end{bmatrix}$                       E. NOTA

21. Matrix  $A$  is multiplied by its multiplicative inverse. The result is

- A.  $I$                       B.  $A^{-1}$                       C.  $2A$                       D.  $A^2$                       E. NOTA

22. Using Cramer's Rule to solve a system of equations involving variables  $a, b, c,$  and  $d,$  the variable  $c$  is found to equal  $\frac{0}{0}$ . This system has

- A. No solutions                      B. One solution, an imaginary number                      E. NOTA  
C. Infinite solutions                      D. One solution, not necessarily an imaginary number

23. Translate the system  $\begin{aligned}3x + 2y &= 5 \\2x - 4y &= 5\end{aligned}$  into a matrix equation.

- A.  $\begin{bmatrix} 3x \\ 2x \end{bmatrix} + \begin{bmatrix} 2y \\ -4y \end{bmatrix} = 5$                       B.  $\begin{bmatrix} -4x \\ 2y \end{bmatrix} + \begin{bmatrix} 2x \\ 3y \end{bmatrix} = 5$                       E. NOTA  
C.  $\begin{bmatrix} 3x \\ 2x \end{bmatrix} \begin{bmatrix} 2y \\ -4y \end{bmatrix} = \begin{bmatrix} 5 \\ 5 \end{bmatrix}$                       D.  $\begin{bmatrix} 3 & 2 \\ 2 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 5 \end{bmatrix}$

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24. Evaluate  $\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}^{11}$ .

- A.  $\begin{bmatrix} 16 & 0 \\ 0 & -16 \end{bmatrix}$       B.  $\begin{bmatrix} 0 & 16 \\ -16 & 0 \end{bmatrix}$       C.  $\begin{bmatrix} -32 & -32 \\ 32 & -32 \end{bmatrix}$       D.  $\begin{bmatrix} 20 & 0 \\ -20 & -20 \end{bmatrix}$       E. NOTA

25. Suppose the  $3 \times 3$  matrix  $A$  has a determinant of 3. What is the determinant of  $3A$ ?

- A. 9      B. 27      C. 81      D. 243      E. NOTA

26. What is the element in the second row, third column of the inverse of the matrix  $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 1 & 0 & 6 \end{bmatrix}$ ?

- A.  $-\frac{5}{22}$       B.  $\frac{1}{11}$       C.  $-\frac{6}{11}$       D.  $\frac{5}{22}$       E. NOTA

27. Evaluate  $\begin{bmatrix} 1 & 2 \\ -4 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -2 & 2 \\ 1 & 3 \end{bmatrix}$ .

- A.  $\begin{bmatrix} -9 & 13 \\ 14 & -20 \end{bmatrix}$       B.  $\begin{bmatrix} -9 & 13 \\ 16 & -35 \\ -4 & 24 \end{bmatrix}$       C.  $\begin{bmatrix} -9 & 13 & -4 \\ 14 & -20 & 24 \end{bmatrix}$       D.  $\begin{bmatrix} -9 & 13 & 16 \\ 14 & -20 & -35 \\ -4 & 24 & 0 \end{bmatrix}$       E. NOTA

28.  $\begin{vmatrix} 3 & 2 \\ -3 & a \end{vmatrix} = \begin{vmatrix} -1 & 3 \\ 6 & 6 \end{vmatrix}$ , what is the value of  $a$ ?

- A.  $\frac{7}{3}$       B. -10      C.  $\frac{11}{3}$       D. 5      E. NOTA

29. If  $\begin{vmatrix} 2 & -3 & 1 \\ x & 1 & 2 \\ 0 & -2 & x \end{vmatrix} = 20$ , find value or the sum of the values of  $x$ .

- A. -2      B. -1      C. 0      D. 2      E. NOTA

30. Find the inverse of  $\begin{bmatrix} i & \pi \\ e & 1 \end{bmatrix}$ .

- A.  $\begin{bmatrix} 1 & \pi \\ e & i \end{bmatrix}$       B.  $\begin{bmatrix} 1 & -\pi \\ -e & i \end{bmatrix}$       C.  $\frac{1}{i-e\pi} \begin{bmatrix} -1 & \pi \\ e & -i \end{bmatrix}$       D.  $\frac{1}{e\pi-i} \begin{bmatrix} -1 & \pi \\ e & -i \end{bmatrix}$       E. NOTA

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Tiebreakers:

1. How many directed paths in the matrix  $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 0 & 3 \\ 2 & 1 & 0 \end{bmatrix}$ .

2. Simplify:  $[3 \ -2 \ 5] \cdot \begin{bmatrix} 4 \\ 6 \\ 10 \end{bmatrix}$

3. If  $\begin{bmatrix} -2 & 1 & 0 \\ 5 & -3 & 1 \end{bmatrix} \cdot \begin{bmatrix} a & 0 \\ 3 & b \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} -3 & 10 \\ 8 & -31 \end{bmatrix}$ , find the sum of  $a + b$ .