2009 Quadratic Topic Test

Solutions

1. $(5x + 3)(x+ 2) < 0$; $x \in (-2, -{}^{3}/_{5})$; $-22/5 \neq x$ ***D***

2. $4x^2 + 4y^2 = 16$
   $\underline{-4x^2 - 9y^2 = -36}$
   $y^2 = 4 \rightarrow y = \pm 2$; $x = 0$; (0, 2) and (0, -2); $ab + cd = 0$. ***B***

3. $x^2 + \sqrt{3}x = -\frac{x}{4}$; $4x^2 + 4\sqrt{3}\,x + x = 0$; $x(4x + 4\sqrt{3} + 1) = 0$; $x = 0$, $x = \frac{-1-4\sqrt{3}}{4}$. ***E***

4. $y^2 + ay + 6 = (y - b)(y + 3) + 1$; $y^2 + (3 - b)y - 3b + 1$; $a = 3 - b$, $6 = 1 - 3b \rightarrow b = {}^{-5}/_{3}$; $a = {}^{14}/_{3}$;
   $2a + b = {}^{28}/_{3} - {}^{5}/_{3} = {}^{23}/_{3}$. ***E***

5. $\frac{5}{2x} + \frac{2}{x-1} = \frac{3x+2}{2x^2-2x}$; $\frac{5x-5+4x}{2x(x-1)} = \frac{3x+2}{2x(x-1)}$; $9x - 5 = 3x + 2$; $x = {}^{7}/_{6}$. ***B***

6. s = side, $s^2 - (s - 2)^2 = 24$; $s^2 - s^2 + 4s = 28 \rightarrow s = 7$; $P = 28$. ***C***

7. $3(1)^2 - 4a(1) - 4(1) - 3 = 4$; $-4a = 8$; $x = -2$. ***A***

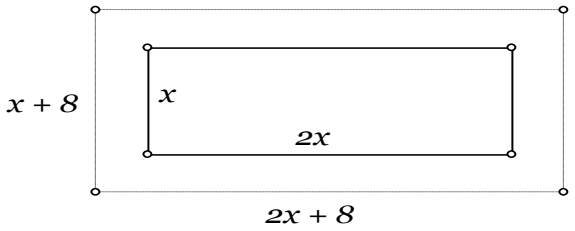
8. $x \nabla y = 2(3 \nabla 1) - (3 \nabla 1)^2$; $2(3 - 1) - [3 - 1]^2 = 0$. ***C***

9. $3^{x^2-10} = 9^{2y}$; $x^2 - 10 = 4y$, $2^{x+y} = \sqrt{2}$; $x + y = ½$; $y = ½ - x \rightarrow x^2 - 10 = 2 - 4x$; $x^2 + 4x - 12 = 0$
   $(x - 2)(x + 6) = 0$; $x = 2, -6$. ***B***

10. $x^2 + x - 2 < 0$. $(x + 2)(x - 1) < 0$; $x \in (-2, 1)$ so -2 not a solution. ***A,D***

11. x = width
    $2x$ = length

    $(x + 8)(2x + 8) = 2880$
    $x = 32$, $2x = 64$
    Length of pool = 64 ***C***



12. $x^2 - y^2 = 9$
    $\underline{-x^2 + 4y = -9}$
    $y^2 - 4y = 0$; $y = 0, 4 \rightarrow (-5, 4),(5, 4),(-3, 0).(3, 0)$; 8. ***D***

13. $3x^2 - 5x - 2 = (3x + 1)(x - 2) \rightarrow x = \frac{-\frac{1}{3}+2}{2} = {}^{5}/_{6}$. ***C***

14. c = $\sqrt{16+20}$ = 6. The length of the median to the hypotenuse of a right triangle is ½ the the length of the hypotenuse so $x = 3$. ***A***

15. $x^2 + 3x - \sqrt{x^2+3x} - 6 = 0$. Let $a = \sqrt{x^2+3x}$, then $a^2 - a - 6 = 0$ and $a = 3, -2$. $\sqrt{x^2+3x} = 3$; $x^2 + 3x - 9 = 0$ but $\sqrt{x^2+3x} \neq -2$, $x_1 + x_2 = -3$. ***D***

16. $2n - 3 = -1, 1, 3, \ldots, 100$, $Sum_1 = -1$, $Sum_2 = 0$, $Sum_3 = 3$, $Sum_4 = 8, \ldots$ -1, 0, 3, 8, 15, … $Sum_n = n^2 - 2n$. $100^2 - 200 = 9800$. ***B***

17. $|x - 5| = 0$, x = 5, then $(y + 3)^2 = 0$ and $y = -3$; $5^2 - 3(-3) = 34$. ***D***

18. $\frac{(x+y)(x-y)}{(x-y)(x-y)} \cdot \frac{x^2 - xy + y^2}{(x-y)(x-y)} \cdot \frac{(x-y)^4}{(x+y)(x^2-xy+y^2)} = \frac{(x-y)}{1} \rightarrow x - y$. ***B***

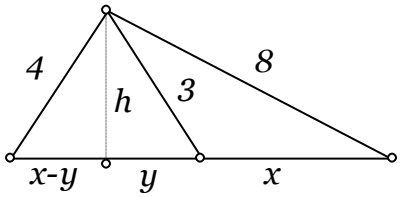
19. $2x(kx - 4) - x^2 + 6 = 0 \rightarrow (2k - 1)x^2 - 8x + 6 = 0$; $b^2 - 4ac = 0$; $64 - 4(6)(2k - 1) < 0$; $k > {}^{11}/_{6}$; 2. ***C***

20. $x^4 - 37x^2 = -36$, since $x^4 - 37x^2 + 36 = 0$ indicates the product of the roots is positive. ***D***

21. $f(½)$ is the value of $g(x)$. $½ = 1 - x^2$; $x = \pm\frac{\sqrt{2}}{2}$; $f\left(\pm\frac{\sqrt{2}}{2}\right) = \frac{1/2}{1/2} = 1$. ***B***

22. $a + b = 10$, $ab = 20$; ${}^{1}/_{a} + {}^{1}/_{b} = {}^{(a+b)}/_{ab} = {}^{10}/_{20} = ½$. ***B***

23. $16 - (x - y)^2 = 64 - (x + y)^2$; $4xy = 48$. $16 - (x - y)^2 = 9 - y^2$; $7 = x^2 - 2xy$; $x^2 - 24 = 7$; $x = \sqrt{31}$; $BC = 2\sqrt{31}$. ***B***



24. $b^2 - 4ac = 16 - 24k > 0$; $k < {}^{2}/_{3}$. ***A***

25. $4x + 11 \geq 21$; $x \geq {}^{5}/_{2}$. ***D***

26. $\frac{ax^2+2ax+a}{x^2+2x+1} = 6$; $\frac{a(x^2+2x+1)}{x^2+2x+1} = a = 6$. ***C***

27. $f(x + a) = 3(x - a)^2 - 2(x + a) + 5$; $3x^2 + x(-2 - 6a) + 3a^2 - 2a + 5 = 3x^2 + 28x + 70$; $-2 + 6a = 28$; $a = 5$; also $3a^2 - 2a + 5 = 70$; $a = 5$. ***C***

28. $\sqrt{10+\sqrt{10+\sqrt{10+\cdots}}} = \sqrt{x} = x - 10$; $x^2 - 21x - 100 = \frac{21 \pm \sqrt{421-400}}{2} = \frac{21+\sqrt{21}}{2}$; $a + b + c = 44$. ***E***

29. Let x and $88 - x$ be the 2 numbers. Then $\frac{(88-x)}{x} = 5 + \frac{10}{x}$; $88 - x = 5x + 10$; $x = 13$, $88 - x = 75$. Product of $75 \cdot 13 = 975$. ***A***

30. $h(t) = -16t^2 + 32t + 3000$. $-16(t^2 - 2t + 1) + 3016 = h(t)$; $t = 1$sec. and $-16 + 32 = 16$ *ft* ***B***.

Tie-Breakers:

1. $(x^3 + x) + 2(x^4 + 2x^2 + 1) = x(x^2 + 1) + 2(x^2 + 1)^2 \rightarrow$ **$(x^2 + 1)(2x^2 + x + 2)$.**

2. $x^4 + 64 = (x^4 + 16x^2 + 64) - 16x^2 \rightarrow$ **$(x^2 + 4x + 8)(x^2 - 4x + 8)$.**

3. $r_1 + r_2 = -4$; $2(r_1 + r_2) = -8$, $r_1 r_2 = -5$; $2r_1 r_2 = -10$; $\rightarrow$ **$x^2 - 8x - 10 = 0$.**