

2009 Radicals Theta Topic Test

Solutions:

1. $x\sqrt{24x^3} + \sqrt{48x} - \sqrt{54x^5} \Rightarrow 2x^2\sqrt{6x} + 4\sqrt{3x} - 3x^2\sqrt{6x} \Rightarrow 4x\sqrt{3x} - x^2\sqrt{6x}$ 1. d
2. $(5)^{\frac{1}{2}}(2)^{\frac{1}{2}}(3)^{\frac{1}{2}} \Rightarrow (30)^{\frac{1}{2}} \Rightarrow \sqrt{30}$ 2. b
3. $\frac{\sqrt{2}-3}{\sqrt{2}+1} \cdot \frac{\sqrt{2}-1}{\sqrt{2}-1} \Rightarrow \frac{5-4\sqrt{2}}{1} \Rightarrow 5-4\sqrt{2}$ 3. a
4. $x^3 - 16 \Rightarrow (x - 2\sqrt[3]{2})(x^2 + 2x\sqrt[3]{2} + 4\sqrt[3]{4})$ 4. b
5. $\frac{\sqrt{a}-\sqrt{b}}{-(\sqrt{a}-\sqrt{b})} = -1 \Rightarrow -1 = -1 \Rightarrow \sqrt{a} \neq \sqrt{b}$ 5. d
6. $\sqrt{9x^4 + \frac{9x^4}{y^2}} \Rightarrow \frac{\sqrt{9x^4y^2 + 9x^6}}{y} \Rightarrow \frac{3x^2\sqrt{y^2+1}}{y}$ 6. d
7. $\frac{3\sqrt[4]{81x^4}}{2} + \frac{2x}{3} \Rightarrow \frac{9x}{2} + \frac{2x}{3} \Rightarrow \frac{31x}{6}$ 7. c
8. $\left(\frac{1}{4} - \frac{1}{9}\right)^{-\frac{1}{2}} \Rightarrow \left(\frac{5}{36}\right)^{-\frac{1}{2}} \Rightarrow \left(\frac{36}{5}\right)^{\frac{1}{2}} \Rightarrow \frac{6\sqrt{5}}{5}$ 8. d
9. $8 + 24\sqrt{x}(2 + \sqrt{x}) = 19(2 + \sqrt{x}) \Rightarrow 8 + 48\sqrt{x} + 24x = 38 + 19\sqrt{x}$ 9. a
 $29\sqrt{x} = 30 - 24x \Rightarrow 881x = 900 - 1440x + 576x^2 \Rightarrow 900 - 2321x + 576x^2$
 $x = \frac{4}{9}, \frac{225}{64} \Rightarrow x \neq \frac{225}{64} \Rightarrow x = \frac{4}{9}$
10. $\frac{\sqrt{\frac{x}{y^3}}}{\sqrt{\frac{y}{x^3}}} \Rightarrow \sqrt{\frac{x^4}{y^4}} \Rightarrow \frac{x^2}{y^2}$ 10. b
11. $\pm .5 = \sqrt[3]{.02x - .1} \Rightarrow \pm .125 = .02x - .1 \Rightarrow x = -1.25 \text{ or } 11.25$ 11. c
12. $f(x-1) = \sqrt{x} \Rightarrow g(\sqrt{x}) = \sqrt{\sqrt{x}} \Rightarrow g(\sqrt{\sqrt{x}}) \Rightarrow \sqrt{\sqrt{\sqrt{x}}} \Rightarrow \left((x^{\frac{1}{2}})^{\frac{1}{2}}\right)^{\frac{1}{2}} \Rightarrow \sqrt[3]{x}$ 12. a
13. $\sqrt{x+1} = 2 - \sqrt{x} \Rightarrow x+1 = 4 - 4\sqrt{x} + x \Rightarrow 3 = 4\sqrt{x} \Rightarrow x = \frac{9}{16}$ 13. d
14. $f(20736^{\frac{1}{8}}) \Rightarrow (2^8 \cdot 3^4)^{\frac{1}{8}} \Rightarrow 2 \cdot 3^{\frac{1}{2}} \Rightarrow 2\sqrt{3}$ 14. c

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$$15. \sum_3^{11} \frac{1}{\sqrt{n+1} + \sqrt{n}} = \frac{1}{\sqrt{4} + \sqrt{3}} + \frac{1}{\sqrt{5} + \sqrt{4}} + \dots + \frac{1}{\sqrt{11} + \sqrt{10}} + \frac{1}{\sqrt{12} + \sqrt{11}} \Rightarrow$$

$$\sqrt{4} - \sqrt{3} + \sqrt{5} - \sqrt{4} \dots \sqrt{11} - \sqrt{10} + \sqrt{12} - \sqrt{11} \Rightarrow -\sqrt{3} + 2\sqrt{3} \Rightarrow \sqrt{3}$$
15. d

$$16. x \left(2x^{3/2} \right) + x^2 + x(3x) \Rightarrow 2x^2 \sqrt{x} + 4x^2 \Rightarrow 2x^2(\sqrt{x} + 2)$$
16. b

$$17. \frac{\sqrt{2} - \sqrt{3} - 1}{(\sqrt{2} - \sqrt{3})^2 - 1} + \frac{\sqrt{2} - \sqrt{3} + 1}{(\sqrt{2} - \sqrt{3})^2 - 1} \Rightarrow \frac{2\sqrt{2} - 2\sqrt{3}}{4 - 2\sqrt{6}} \Rightarrow \frac{\sqrt{2} - \sqrt{3}}{2 - \sqrt{6}} \cdot \frac{2 + \sqrt{6}}{2 + \sqrt{6}} \Rightarrow \frac{2\sqrt{2} - 2\sqrt{3} + \sqrt{12} - \sqrt{18}}{-2}$$

$$\frac{-\sqrt{2}}{-2} \Rightarrow \frac{\sqrt{2}}{2}$$
17. d

$$18. x = \frac{\sqrt{3} \pm \sqrt{5+72}}{2} \Rightarrow \frac{\sqrt{3} \pm 5\sqrt{3}}{2} \Rightarrow 3\sqrt{3} \text{ or } -2\sqrt{3}$$
18. d

$$19. \sqrt{y+5} = 5 - \sqrt{8-y} \Rightarrow y+5 = 25 - 10\sqrt{8-y} + 8 - y \Rightarrow 2y - 28 = -10\sqrt{8-y} \Rightarrow$$

$$y - 14 = -5\sqrt{8-y} \Rightarrow y^2 - 28y + 196 = 200 - 25y \Rightarrow y^2 - 3y - 4 = 0 \Rightarrow y = 4 \text{ or } -1$$

Sum = 3

19. c

$$20. (2^3 c^2 d^4)^{2/3} \cdot (3^2 c^3 d^3)^{1/2} \Rightarrow 2d(c^{2/3} d^{1/3}) \cdot 3cd(c^{1/2} d^{1/2}) \Rightarrow 6cd^2(c^{7/6} d^{5/6}) \Rightarrow 6c^2 d^2 \sqrt[6]{cd^5}$$
20. b

$$21. r_1 + r_2 = 4 \text{ and } r_1 \cdot r_2 = -50 \Rightarrow x^2 - 4x - 50 = 0$$
21. b

$$22. (2\sqrt{3} + \sqrt{2} - 3)(\sqrt{3} - 2\sqrt{2} + 1) \Rightarrow (6 - 4\sqrt{6} + 2\sqrt{3} + \sqrt{6} - 4 + \sqrt{2} - 3\sqrt{3} + 6\sqrt{2} - 3)$$

$$-1 + 7\sqrt{2} - \sqrt{3} - 3\sqrt{6}$$
22. d

$$23. \sqrt{x+1} = \sqrt{x-3} + 1 \Rightarrow x+1 = x-3 + 2\sqrt{x-3} + 1 \Rightarrow 3 = 2\sqrt{x-3} \Rightarrow \frac{9}{4} + 3 = x$$

$$x = \frac{21}{4} \Rightarrow y = 2.5$$
23. b

$$24. \sqrt{7+2\sqrt{y}} - \sqrt{7-2\sqrt{y}} = 2 \Rightarrow 7+2\sqrt{y} - 2\sqrt{49-4y} + 7-2\sqrt{y} = 4 \Rightarrow 10 = 2\sqrt{49-4y}$$

$$5 = \sqrt{49-4y} \Rightarrow 25 = 49 - 4y \Rightarrow y = 6$$
24. a

$$25. \sqrt{x+2\sqrt{y}} - \sqrt{x-2\sqrt{y}} = 2 \Rightarrow x+2\sqrt{y} - 2\sqrt{x^2-4y} + x-2\sqrt{y} = 4 \Rightarrow 2x-4 = 2\sqrt{x^2-4y}$$

$$x^2 - 4x + 4 = x^2 - 4y \Rightarrow y = x - 1 \Rightarrow (6, 5)$$
25. d

$$26. 18.2 = v_o \sqrt{\frac{3.6}{4.9}} \Rightarrow 18.2 \cdot \sqrt{\frac{4.9}{3.6}} = v_o \Rightarrow v_o = 21.2$$
26. e

$$27. s = m\sqrt{K} \Rightarrow \frac{320}{100} = \frac{16}{\sqrt{K}} \Rightarrow K = 25^0$$
27. a

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$$28. \quad (2 + 3\sqrt{7})(a + b\sqrt{7}) = -107 - 13\sqrt{7} \Rightarrow 2a + 21b + \sqrt{7}(3a + 2b) = -107 - 13\sqrt{7} \Rightarrow$$

$$2a + 21b = -107 \text{ and } 3a + 2b = -13 \Rightarrow (-1, -5) \Rightarrow -6$$

28. a

$$29. \quad x = \frac{3 \pm \sqrt{9 - 4c}}{2} \Rightarrow 9 - 4c \neq n^2 \Rightarrow 9 - 4c^2 \neq 0, 1, 4, 9, 16, \dots \Rightarrow c \neq -\frac{7}{4}, \frac{5}{4}, \frac{9}{4} \Rightarrow c = \frac{7}{4}$$

Omit

$$30. \quad x = \sqrt{10 + \sqrt{10 + \sqrt{10 + \dots}}} \Rightarrow x^2 = 10 + x \Rightarrow x = \frac{1 + \sqrt{41}}{2} \Rightarrow b = 41$$

30. d

Tie-Breakers:

$$1. \quad \sqrt{13 - 4\sqrt{3}} - \sqrt{31 + 12\sqrt{3}} + \sqrt{12 - 6\sqrt{3}} \Rightarrow \sqrt{(1 - 2\sqrt{3})^2} - \sqrt{(2 + 3\sqrt{3})^2} + \sqrt{(3 - \sqrt{3})^2} \Rightarrow$$

$$1 - 2\sqrt{3} - 2 - 3\sqrt{3} + 3 - \sqrt{3} \Rightarrow 2 - 6\sqrt{3}$$

$$2. \quad (2)^2 - 3\sqrt{2}(2) + c = 0 \Rightarrow c = -4 + 6\sqrt{2}$$

$$3. \quad y = 2\sqrt{x+3} - 2$$