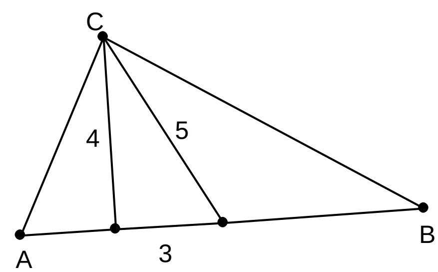
2009 Triangle Topic Test (Theta)

Solutions

<table>
<tr><td>1.</td><td colspan="2">$Area = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{18(8)(6)(4)} = 24\sqrt{6}$ **D**</td></tr>
<tr><td>2.</td><td colspan="2">Hypotenuse of first right triangle = $\sqrt{5^2+10^2} = \sqrt{125} = 5\sqrt{5}$. Ratio of similitude 1st rt. Δ to 2nd rt. Δ is $\frac{5\sqrt{5}}{15} = \frac{\sqrt{5}}{3}$, which means that the ratio of the areas is $\left(\frac{\sqrt{5}}{3}\right)^2 = \frac{5}{9}$;
$\frac{A_1}{A_2} = \frac{\frac{1}{2}(5)(10)}{A_2} = \frac{25}{A_2}$; $\frac{25}{A_2} = \frac{5}{9}$; $A_2 = \frac{(25)(9)}{5} = 45$ **C**</td></tr>
<tr><td>3.</td><td>Area of the triangle is irrelevant. The projection, altitude, and median form a right triangle whose legs are the projection and altitude, lengths 3 and 4 respectively. The median is the hypotenuse which must be length 5.

B</td><td>
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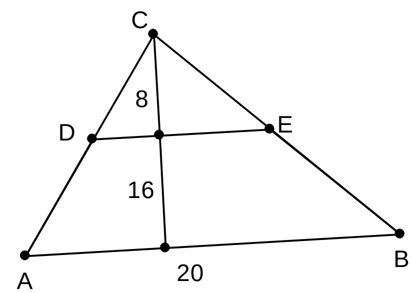
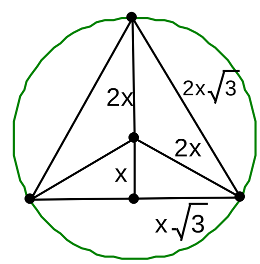
<tr><td>4.</td><td>$area = \frac{1}{2}bh$; $240 = \frac{1}{2}(20)h$; $h = 24$
$\Delta CDE \sim \Delta CAB$ with ratio $\frac{8}{24} = \frac{1}{3}$. Ratio of areas $\left(\frac{1}{3}\right)^2 = \frac{1}{9}$
Therefore the area of the trapezoid ABED is $\frac{8}{9}$ the area of triangle ABC. $\frac{8}{9}(240) = \frac{8}{3}(80) = \frac{640}{3}$ **A**</td><td>
</td></tr>
<tr><td>5.</td><td>$A = \pi r^2$
$196\pi = \pi(2x)^2$
$196\pi = \pi(4x^2)$
$\sqrt{49} = 7 = x$
$3x = 21$

B</td><td>
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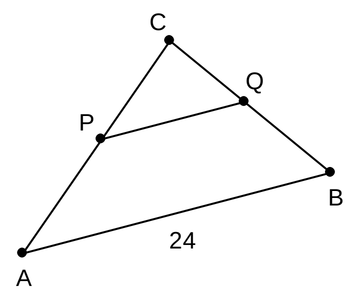
<tr><td>6.</td><td>$\frac{area\Delta CPQ}{area\Delta CAB} = \left(\frac{PQ}{AB}\right)^2 = \left(\frac{1}{3}\right) \Rightarrow \frac{PQ}{AB} = \frac{1}{\sqrt{3}}$ **C**

$\frac{PQ}{24} = \frac{1}{\sqrt{3}} \Rightarrow PQ = \frac{24}{\sqrt{3}} = 8\sqrt{3}$</td><td>
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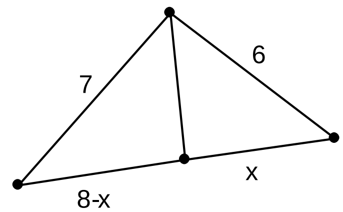
<td>7.</td>
<td>$\frac{7}{6} = \frac{8-x}{x} \Rightarrow \quad 7x = 48 - 6x \Rightarrow \quad 13x = 48 \Rightarrow \quad x = \frac{48}{13}$

D

$8 - x = \frac{56}{13}$</td>
<td></td>
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<tr>
<td>8.</td>
<td colspan="2">Perimeter of triangle ABC is 15.
$\frac{A_1}{A_2} = \left(\frac{perimeter_1}{perimeter_2}\right)^2 = \left(\frac{1}{3}\right) \Rightarrow \quad \frac{perimeter_1}{perimeter_2} = \frac{1}{\sqrt{3}} \Rightarrow \quad \frac{15}{perimeter_2} = \frac{1}{\sqrt{3}} \Rightarrow \quad perimeter_2 = 15\sqrt{3}$ **C**</td>
</tr>
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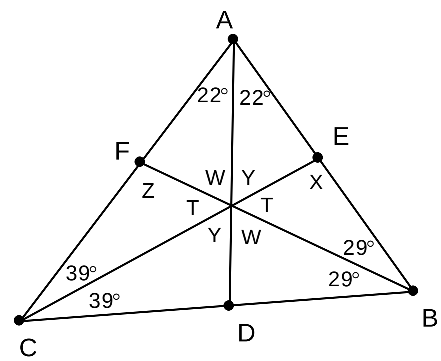
<td>9.</td>
<td>Given that $m\angle BAC = 44°$ and $m\angle ABC = 58°$ it follows that $m\angle ACB = 78°$ and $39° + x + 58° = 180°$ *and* $x = 83°$
$x = y + 22° \Rightarrow 83° = y + 22° \quad \Rightarrow \quad y = 61°$

$z = 180° - 29° - 78° = 73°$
$z = w + 22° \Rightarrow 73° = w + 22° \quad \Rightarrow \quad w = 51°$

$w + y + t = 180° \Rightarrow 51° + 61° + t = 180° \quad \Rightarrow \quad t = 68°$

D</td>
<td></td>
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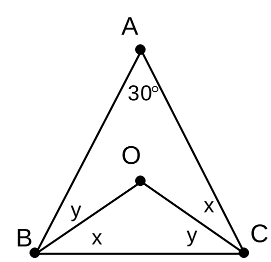
<td>10.</td>
<td>*Since* ΔABC *is isosceles, its base angles are each* 75°
$x + y = 75° = m\angle ABC = m\angle ACB$
$x + y + m\angle BOC = 180°$
substition gives
$75° + m\angle BOC = 180°$
$m\angle BOC = 180 - 75 = 105°$

C</td>
<td></td>
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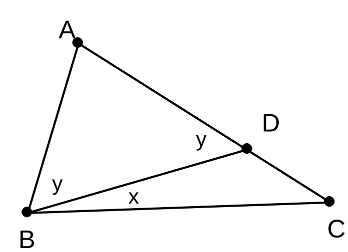
<td>11.</td>
<td>x + y –C = 20 and y = x + C (Exterior angle = sum of remote interior angles)

Substituting x + C for y in the first equation gives
x + x + C - C = 20
2x = 20
x = 10

A</td>
<td></td>
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<td>12.</td>
<td colspan="2">$a^2 + b^2 = c^2 \quad (note: c = a + 1)$
$b^2 = c^2 - a^2 = (a+1)^2 - a^2 = a^2 + 2a + 1 - a^2 = 2a + 1$
$a + c = 2a + 1$ **B**</td>
</tr>
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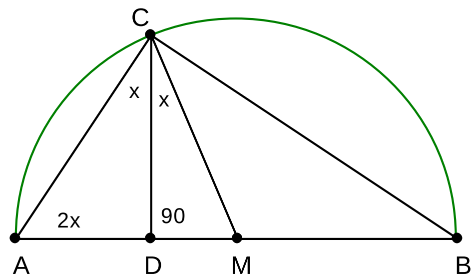
<tr><td>13.</td><td>Given right ΔABC with $m\angle MCD = \frac{1}{2} m\angle A$
MA = MC = MB, (radii of the circle our Δ can be inscribed in) meaning that Δ AMC is isosceles. Since base angles of an isosceles triangle are congruent, $m\angle$ MCA must = 2x and $m\angle$ DCA = x. Since $\angle$ BMC is an exterior angle to Δ AMC and Δ DMC the following equation is true.
2x + 2x = 90 + x = $m\angle$ BMC 3x = 90 and x = 30. C</td><td>
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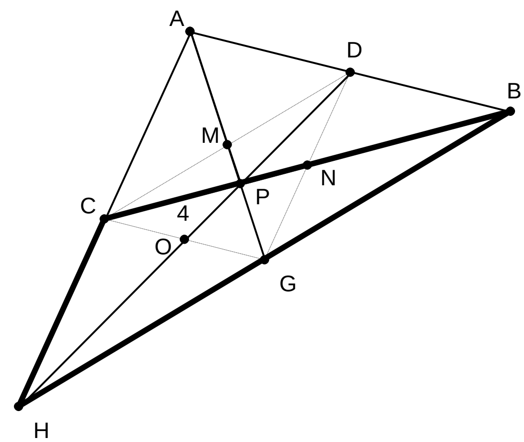
<tr><td>14.</td><td>As seen in the sketch extending AC its own length to H so that C is a midpoint and drawing BH creates triangle ABH. Draw triangle formed by the mid-segments of triangle ABH. The centroid of the mid-segment triangle is also the centroid of the original triangle. (Median of Triangle always contains midpoint of mid segment that it intersects.) Therefore P is the centroid of both triangle ABH and CDG. And therefore PB = 8 and CB = 12.

D</td><td>
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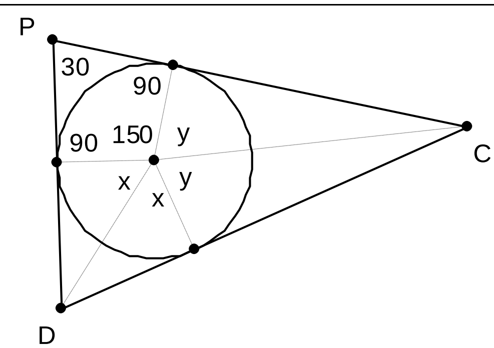
<tr><td>15.</td><td>2x + 2y + 150 = 360
2x + 2y = 210
x + y = 105

A</td><td>
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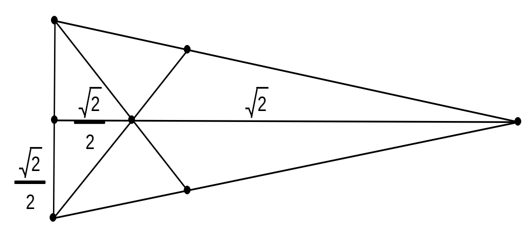
<tr><td>16.</td><td>Altitude (Median) to base bisects the 90 degree angle formed by medians to form two congruent 45-45-90 triangles along the base.

Area = $\frac{\sqrt{2}}{2}\left(\frac{\sqrt{2}}{2}+\sqrt{2}\right)=\frac{\sqrt{2}}{2}\left(\frac{3\sqrt{2}}{2}\right)=\frac{3(2)}{4}=\frac{3}{2}$ A</td><td>
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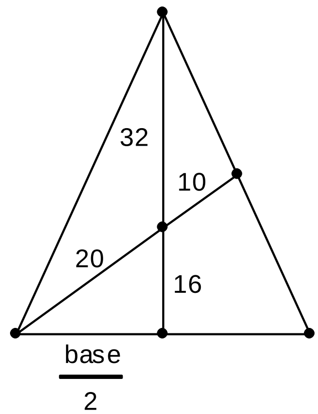
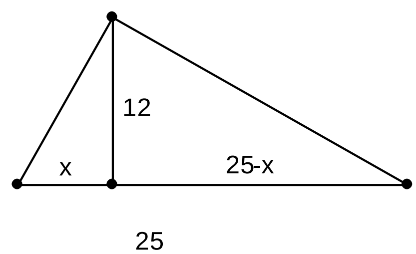
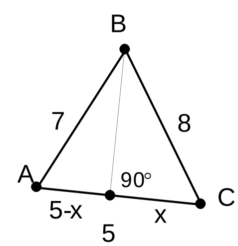
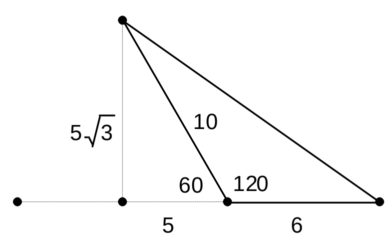
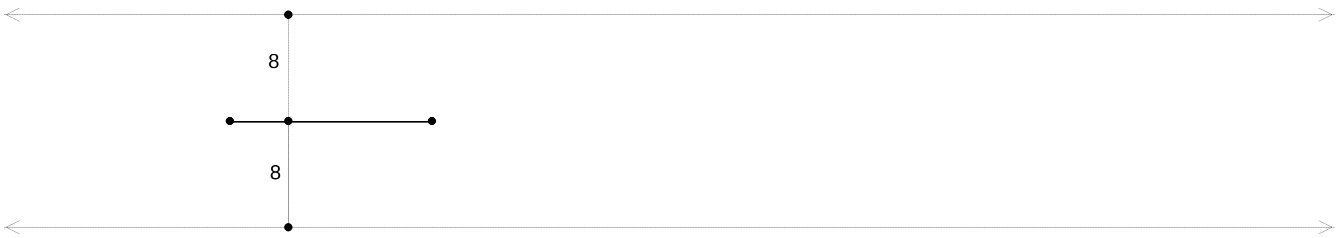
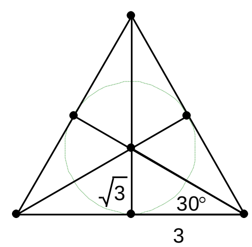
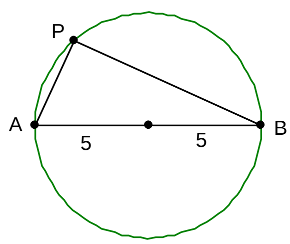
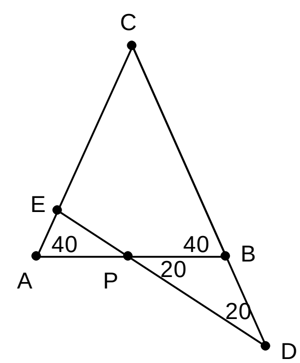
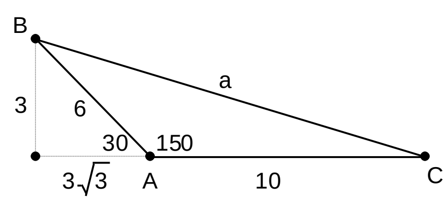
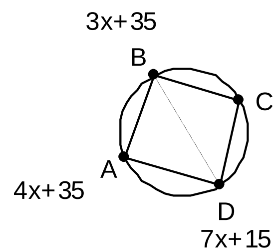
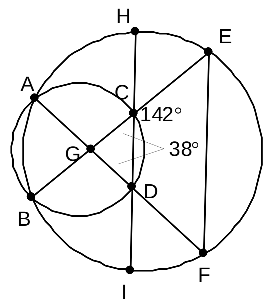
17.	The triangle must be isosceles since two of the medians are equal in measure. Therefore the median that is 48 in length must also be the altitude of the triangle. As shown in the sketch, a multiple of 3-4-5 right triangle is formed in which $\frac{base}{2}=12$. Therefore the area of the triangle is $\frac{base}{2}(h)=12(48)=576.$ **C**	
18.	Since this is an isosceles right triangle the altitude to the hypotenuse is also half the hypotenuse .5(10)=5 **E**	
19.	$(x)(25-x)=12^2$ $25x-x^2=144$ $x^2-25x+144=0$ $(x-16)(x-9)=0$ $x=16 \quad x=9$ **C**	
20.	Let A = (-2, 1), B = (2,4), C = (3,1). AB = $\sqrt{4^2+3^2}=5$ $BC=\sqrt{1^2+3^2}=\sqrt{10}$ $AC=\sqrt{5^2+0^2}=5$ **C** Not a right triangle since slopes of sides are $\frac{3}{4}, -3, \& \ 0$.	
21.	$7^2-(5-x)^2=8^2-x^2$ $49-25+10x-x^2=64-x^2$ $24+10x=64$ $10x=40$ $x=4$ **B**	
22.	Area = .5 (6) ($5\sqrt{3}$) Area = $15\sqrt{3}$ **D**	
23.	30, 40, 50 is a right triangle whose area is 600 and value of s = 60. radius = 600/60 = 10. **D**	
24.	Locus is the center of the inscribed circle. Distance would be the radius of the inscribed circle which is	

	equal to the area of the triangle divided by the semi-perimeter = $\dfrac{\frac{1}{2}(3)(4)}{6}=\dfrac{6}{6}=1$	**A**
25.	The angle bisectors of a triangle are concurrent in a point called the in-center.	**A**
26.	H = 8, therefore any point a distance of 8 from the line containing AB is on the locus.  	**D**
27.	Since this triangle is equilateral, each altitude is also a median and angle bisector. **C**  	
28.	 **B**	
29.	Triangles ABC and BDP are isosceles. BP = BD and AC = BC Since base angles of isosceles triangles are equal, $m\angle D = m\angle BPD$ Exterior angle ABC = to the sum of $m\angle D \,\&\, m\angle BPD = 40$ Again, since base angles of isosceles triangles are equal, $m\angle A = m\angle CBA$ **D**  	
30.	$a^2 = 3^2 + \left(10+3\sqrt{3}\right)^2$ $a=\sqrt{9+100+60\sqrt{3}+27}$ $a=\sqrt{136+60\sqrt{3}}$ **E**  	

31. TB1	3x + 35 + 7x+ 15 = 180 and x = 13 4x + 35 + C = 180 52 + 35 + C = 180 and C = 93° which must be the measure of the largest angle in triangle BCD	
32. TB2	$m\angle AGE = sum\ \ remote\ \ angles = 76°$	
33. TB3	By Ceva's Theorem $(x)(2)(z)=(3)(y)(1)$ *and then* $\frac{(x)(z)}{(y)}=\frac{3}{2}$