Question #0
Mu Ciphering
2010 MAΘ National Convention

Evaluate: $\int_0^1 2x(1-x^2)^{10}dx$

Question #1
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Let $f(x)=\sum_{n=1}^{\infty}x^n$ for $|x|<1$. Find the maximum value of $\frac{f(x)}{f'(x)}$.

Question #2
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Given that $\lim_{n\to\infty}\left(1+\frac{a}{n}\right)^n=e^a$, let $f(x)=\lim_{n\to\infty}\left(\frac{n+0.5\sin(2x)}{n}\right)^{n\tan(x)}$.

Find $f'\left(\frac{\pi}{6}\right)$.

Question #3
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Giulio and Gerardo are at the origin and decide to part ways. Gerardo walks along the y-axis at 0.4 m/s and Giulio along the x-axis at 0.2 m/s. After one **minute**, at what rate is the distance between them increasing in meters per second?

Question #4
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A triangle has vertices at (0, 0), $(a^2+1, 0)$, and $(3a^3-a^2+2a-4, 5)$. If a is increasing at 3 units/sec, find the rate of change of the triangle's area when $a = 6$.

Question #5
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If $\int_{\sqrt{\ln 2}}^{\sqrt{\ln 3}} x^3e^{x^2}dx = A\ln 3 + B\ln 2 + C$, where A, B, and C are integers, find $A+B+C$.

Question #6
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The smallest prime factor of 2001 is A. The largest prime factor of 2001 is B. Find $A+B$.

Question #7
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If $3x^2+xy^2-y^3\cos(x)+8=0$, find $\frac{dy}{dx}$ at the point (0, 2).

Question #8

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Using L'Hopital's Rule once, compute $\lim_{x\to 0^+}\left[\left(1-\cos(x)\right)^{-1/2}\int_0^{\sin(3x)} e^{2t}dt\right]$.

Question #9

Mu Ciphering
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Every day I pay one dollar to roll a pair of dice. If both dice land on 6, then I get k dollars and never play this game again. If I don't win, then I play again the next day. If my expected winnings for this game are zero, what is k? Assume my immortality.

Question #10

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Suppose n people, including Ms. Herron, leave their hats at the door when entering a party. These same n people go to a party for n days in a row. Because of the vast quantity of people, however, each one simply takes a random hat each night when they leave the party. Let $P(n)$ denote the probability that Ms. Herron never leaves a party with the same hat that she brings. Find $\lim_{n\to\infty} P(n)$.