Question #0
Theta Ciphering
MAΘ National Convention 2010

Factor $20x^2 + 33x - 36$ into $(ax + b)(cx + d)$, such that $a, b, c$ and $d$ are integers, and $a > 0$.
Then, find the value of $a + b + c + d$.

Question #1
Theta Ciphering
MAΘ National Convention 2010

Wayne’s pool is now empty, so he wants to fill it. He opens both faucets. Faucet A, by itself, can fill the pool in 4 hours and faucet B, by itself, can fill the pool in 6 hours. However, ever the lame brain, Wayne forgets to close the drain. This drain can empty the pool in 8 hours. How many hours will it take to completely fill Wayne’s pool if both faucets are on (full) and the drain is completely open? Express your answer as a mixed number with the fractional part in lowest terms.

Question #2
Theta Ciphering
MAΘ National Convention 2010

Determine the minimum number of games necessary in a 251 team tournament in order to determine a champion if the format of the tournament is
S = single elimination (a team is eliminated after one loss)
D = double elimination (a team is eliminated after losing two games).

Find the value of $S \cdot D$.

Question #3
Theta Ciphering
MAΘ National Convention 2010

Express in simplest rational form.
\[
\begin{align*}
&\frac{1}{5} + \frac{1}{6} \\
&\frac{1}{6} + \frac{1}{10} \\
&\frac{1}{1} + \frac{1}{8} \\
&\frac{1}{16} + \frac{4}{8} \\
\end{align*}
\]

Question #4
Theta Ciphering
MAΘ National Convention 2010

Part I. Let $w$ be the radius of a circle with a central angle having measure of $60^\circ$ and its intercepted arc length is $3\pi$.
Part II. Find the values of $x$ and $y$, where in $\triangle ABC$, $m\angle B = 90^\circ, m\angle A = (x + 2y)^\circ$, $m\angle ACB = (2x + y)^\circ$,
and the measure of the exterior angle at $\angle BCA = 125^\circ$.

Find the value of $x + y + w$.

Question #5
Theta Ciphering
MAΘ National Convention 2010

Factor $x^4 - 39x^2 + 49$ into two trinomials with positive leading coefficients.

Question #6
Theta Ciphering
MAΘ National Convention 2010

If $f(x) = x^2 - 2x$, find the sum of all real values for $x$ so that $f(x) = f \circ f(x)$.
Question #7
Theta Ciphering
MAΘ National Convention 2010

Find all ordered pairs, \((a, b)\), that solve the system of equations over the real numbers.

\[
\log_{10}\left(\frac{a^5}{b^3}\right) = 5
\]

\[
\log_{10}(a^3b^4) = 3
\]

Question #8
Theta Ciphering
MAΘ National Convention 2010

Find the distance, in simplest radical form, between the center of

\[36x^2 + 16y^2 - 216x + 160y = -148\]

and the point of intersection of

\[2x - y = 0\]

and

\[x - y = -4\].

Question #9
Theta Ciphering
MAΘ National Convention 2010

Find the length of \(AB\) when in right \(\triangle ABC\) with \(BD\) the altitude to the hypotenuse with \(D\) on hypotenuse \(AC\), \(BD = 10, CD = 4\).

Question #10
Theta Ciphering
MAΘ National Convention 2010

A giant watermelon weighed 100 pounds and was 99% water by weight in pounds. While standing in the sun, some water evaporated, so that the watermelon was only 98% water by weight in pounds. How many pounds did the watermelon then weigh?