SOLUTIONS

1. C	7. B	13. D	19. A	25. A
2. D	8. D	14. C	20. B	26. D
3. D	9. C	15. C	21. C	27. C
4. D	10. B	16. B	22. A	28. B
5. B	11. B	17. B	23. A	29. B
6. A	12. E	18. A	24. E	30. C

1. C. In one hour, the angle will be

 $\frac{\pi}{180}$ $= \frac{\pi}{3}$ and so the radii create an

equilateral triangle. So the distance between the points is the same as the radii.

<u>2.</u> D. Ball 1: 100 + 25(2) + 25(2)/4 + ... =

$$100 + \frac{50}{1 - \frac{1}{4}} = \frac{500}{3}$$
. Ball 2: $80 + \frac{120}{1 - \frac{3}{4}} = 560$.

| 3R-S | = | 500-560 |= 60.

3. D. Process of elimination, given that Martha (M) and her sister (Sis) are siblings, and her son (S) and daughter(D) are also siblings. If we say M is the <u>best</u>, then D is diagonal from Sis, and we have Sis M

Son D

and so Martha's sister is worst.

If D is best, then she would have to be diagonal from herself. If S is best then same happens. If Sis is best then D is diagonal from but this M and D Sis contradicts Μ

sis being

the son and

on the diagonal. So Sis must be worst.

4. D. When y=1, Tanx=1 and x=pi/4. The graph of tanx also goes through the origin and since tanx has period pi the graph also passes

through (-pi, 0). Distance from $\left(\frac{\pi}{4}, 1\right)$ and

$$(-\pi,0)$$
 is $\sqrt{\frac{25\pi^2}{16}+1}$.

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5. **B**. (2x-1) is positive but (5-3x) and (x-6) are negative. So we add (2x-1)+(3x-5)+(6-x).

6. A. Triangle ADK (for K the vertex above B, is 1/2(8)(8) and the lower rectangle is 3(8). Total 32+24 =56

7. B.
$$\sqrt{4} + \sqrt{x+3} = 3$$
, $4 + \sqrt{x+3} = 9$,
 $\sqrt{x+3} = 5$, $x+3 = 25$, $x = 22$

8. D. Multiply numerator and denominator of the left side by $2 + \cos x$ to get

$$\frac{1}{8+3\cos x} = \frac{2}{19} \text{ and solve to get } \cos x = \frac{1}{2}$$

and
$$x = \frac{\pi}{3}$$
 and 1+3=4.

<u>9.</u> *C*. x can be 10 or -10, y can be 5 or -5. |x-y| is least when x=10 and y=5. |x+y| is least when x=10 and y= -5. 5+5=10.

10. B.
$$\frac{25}{3}, \frac{36}{4}, \frac{49}{5}, \frac{64}{6}$$
 gives x= 81/7.

 $\sqrt{a^2 + b^2}$ gives the next term for consec.

terms a and b. Next is $\sqrt{173}$.

Ans=81+173=254.

11. B. In order from lightest: Jelly, Brandi, Copper, Pumkin, Jam. 3rd heaviest is Copper. 12. E. Since we cross paths, I am the only one going to the City of Lights.

13. D. A half circle of radius 4, a guarter circle of radius 1 and a quarter circle of radius 3.

$$\frac{1}{2}16\pi + \frac{1}{4}(\pi + 9\pi) = 10.5\pi$$
.

14. C. 5 small squares, 5 mediums (the same size as the middle part which has 4 smaller squares inside it. And one big square whose top is the highest segment.

15. C. A is false since 6 has a factor of 3 and 2 does not so the two sides cannot be equal with integer powers. B is wrong, which can be verified by trial and error. D is wrong, as

proved by Andrew Wiles and postulated by Fermat. C is true. Consider a triangular number $\frac{1}{2}n(n+1)$ and its next tri. number $\frac{1}{2}(n+1)(n+2)$. This reduces to a perfect square. 16. B. The points in rectangular form are 16 (cont.) $(-4\sqrt{2}, 4\sqrt{2})$ and $(-4\sqrt{2}, -4\sqrt{2})$ which is a distance of $8\sqrt{2}$. 17. B or B. Since at least two numbers are even, one factor is 4. Since at least one number is div. by 5, and at least one is div. by 3, the product must be div. by 60. 18. A. Let AB=2x+x=3x. So 3x=8 and so AC=16/3 and the shaded region is $\frac{1}{2}\left(16\pi - \frac{64}{9}\pi\right)$ 19. A. The two possible lines are each 8 units from line L1. $\frac{6x - 8y - 12}{+10} = 8$ which gives equations 3x-4y=46 or -34. |46-(-34)|=80 **20. B.** The numbers are 2^3 , $2\square 3$, $2\square 5$ and 8+6+10=24 **21.** C. $\sin A = \sin(2x), \cos B = \cos(2y)$ so A=2x, B=2y. A+B=2x+2y. =2(12)=24. 22. A. For f, the first and last factors are always positive due to even powers. So the only change of sign occurs at x= -2. Try intervals: $f \xrightarrow{-} f \xrightarrow{+} f \xrightarrow{+} f \xrightarrow{+} g$ and the positive values are -1, 0, 1, 2, 4, 5, ... Same reasoning for g: + <u>-</u> <u>-</u> g and the positive values are less than 2. The intersection is -1, 0, 1.

<u>23.</u> A. The period of y=cos(2x) is pi. The abs. value will reflect the part below the x-axis up.

The picture is then and the intersection happens 4 times.



<u>24.</u> E. For x > 0 we solve $x^2 - 5x + 6 = 0$ and x=2 or 3. For x < 0 we solve $x^2 + 5x + 6 = 0$ which gives x= -2, -3. The least is -3. 25. A. If x is the amount we remove, we will remove .2 pink, and replace with x pink. Originally we have 4 grams of pink. So for pink to equal white, we must have 4-0.2x+x = 14-0.7x which solves to x=10/1.5 = 100/15 = 20/3. The fraction of pink will be 4-(.2)(20/3)+20/3 divided by total 20. which gives 7/15. 26. D. Use triangle BCD and the law of cosines. 25+36-2(30)cosC=13 to give cosC=4/5 so sinC=3/5. Use the area formula $\frac{1}{2}(4)(6)(\frac{3}{5})=$ 36/5. **<u>27.</u>** $\frac{1}{6}(3+4+5+6) = \$3$ winnings minus $\frac{2}{6}(1)$ gives 3-0.33 = \$2.67 to nearest cent. 28. B. 15% of 800 = 120 increase which will take 120/\$8 = 15 more hours. **29. B.** At one hour, the hypotenuse is x in each case. One height is $\frac{x}{2}$ and the other $\frac{x\sqrt{3}}{2}$ and the difference $x(\frac{\sqrt{3}-1}{2}) = kx$ so $k=\frac{\sqrt{3}-1}{2}$ 30. C. By disjunctive syllogism, the first two

statements give If x then z and since "not z"

we know by modus tollens that "not x" and "not y" are true. However, we do not know that "not k" is true.