Theta Gemini Solutions

The following were changed at the resolution center at the convention: 14 D, 21 D, 24 A

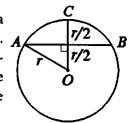
1. D:
$$2^{3^2}$$
 exponents are right associative. $2^9 = 512$
2. C: The distance of $3-4i$ to the origin is 5.
3. B Change of base rule: $\frac{\log_3 9^{\frac{1}{3}}}{\log_3 3^{\frac{1}{2}}} = \frac{\log_3 3^{\frac{2}{3}}}{\log_3 3^{\frac{1}{2}}} = \frac{\frac{2}{3} \log_3 3}{\frac{1}{2} \log_3 3} = \frac{4}{3}$
4. C: $(a+bi)+(a-bi) = 2a$. A: No a could be 0. B: No (no i) D
5. E: use the Harmonic mean $\frac{2}{\frac{1}{40} + \frac{1}{50}} = \frac{400}{9} = 44.4$
6. C $122121 \rightarrow 577 \rightarrow 19$
7. A $77^{5} = 49$ $7^{5} = 343$ $7^{5} = 2401$ cycles 7,9,3,1 42 mod $4 = 2$ 9
 $2^{5}7 = 128$ so $8+9 = 17$ answer 7
8. D repeated div $625/5 = 125$ $125/5 = 25$ $25/5 = 5$ $5/5 = 1$ $125+25+5+1 = 156$
9. A replace x,y and z with 1 $(2+3-3)^{5}10 = 2^{5}10 = 1024$
10. B $\sqrt{34-24\sqrt{2}} = a+b\sqrt{2} \rightarrow 34-24\sqrt{2} = a^{2}+2b^{2}+2ab\sqrt{2} \rightarrow a^{2}+2b^{2} = 34, ab = -12$ by substitution of $b = -\frac{12}{a}$ leads to $a^4 - 34a^2 + 288 = 0$ $a = \pm 4$ or $a = \pm 3\sqrt{2}$ so $a + b\sqrt{c} = 4 - 3\sqrt{2}$ or $-4 + 3\sqrt{2}$ we need the positive answer so $-4 + 3\sqrt{2}$ so $a + b + c = 1$
11. C $9886^{5}2 = (9887-1)^{5}2 = 9887^{5}2 - 2^{2} 9887 + 1 = 97752769 - 2^{2}9887 + 1 = 97732996$

sum of the digits is 52

12. A x = 3.192192... 1000x = 3192.192... subtract 999x = 3189 x = 3189/999 reduce 1063/333 sum 1396

13. A

Draw \overline{AO} , forming a $30^{\circ}-60^{\circ}-90^{\circ}$ right triangle. **A** Since the length of the leg opposite the 60° angle is $3\sqrt{3}$, the length of the radius is 6 and the area of the circle is 36π

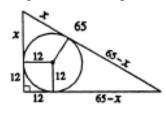


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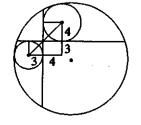
MAO National Convention 2010

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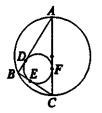
14. C In the diagram, the sides of the square are 12. Let the segments of the hypotenuse be x and 65-x. Since tangents drawn to a circle from the same outside point are congruent, two segments have length x and two have length 65-x. The perimeter of the triangle is 24 + 2(65-x) + 2x = 154



15. C As can be seen in the figure, the distance between the centers is the length of the hypotenuse of an isosceles right triangle with legs 7. $7\sqrt{2}$



16. A Let CF = x. Since the diameter of the circle is 20, AF = 20-x. Tangents to a circle from the same outside point are congruent, so AD=AF=20-x, and CE=CF=x. Since the perimeter of triangle ABC is 42, EB+BD+AD+AF+CF+CE=EB+BD+40=42 so EB+BD = 2



17. B The nth row of Pascal's triangle (start counting at 0) has a sum of 2ⁿ.

18 E Both -2 and -1 must be excluded

19. C
$$(1 \otimes 2) \otimes 3 = (1 + 2 - 3(\frac{1}{2})) \otimes 3 = \frac{3}{2} \otimes 3 = \frac{3}{2} + 3 - 3(\frac{\frac{3}{2}}{3}) = 3$$

20. E: Harmonic Series diverges.

21: E
$$\frac{5!}{2}$$

22. D f(-1) = 9 + 2 -4 -6 -2 - 1 = -2

23. A $\frac{1}{1 \cdot 2 \cdot 3} + \frac{2}{2 \cdot 3 \cdot 4} + \frac{3}{3 \cdot 4 \cdot 5} + \dots + \frac{98}{98 \cdot 99 \cdot 100} = \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \frac{1}{4 \cdot 5} + \dots + \frac{1}{99 \cdot 100}$

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These terms are from a telescoping series

$$\frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1}$$
 so series turns into $\frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \dots + \frac{1}{99} - \frac{1}{99} + \frac{1}{100} = \frac{1}{2} - \frac{1}{100} = \frac{50 - 1}{100} = \frac{49}{100}$ so $a + b = 149$

24. B
$$\begin{vmatrix} 3 & 2 & 7 \\ k & -1 & k \\ 0 & 1 & -4 \end{vmatrix} = -2k + 4$$

 $3*4+7k+0 - (0+3k-8k) = -2k+4 \rightarrow 12 + 7k + 5k = -2k+4 \rightarrow 12+12k = -2k+4 k = -4/7$

- 25. D Center is (4,3) radius is $\sqrt{7}$
- 26. D. $\ln(?)=0 \rightarrow \log_2(\log_3 x)=1 \rightarrow \log_3 x=2 \rightarrow x=9$

27. A
$$\begin{array}{c} 2x+7x=3\\ -2x+2y=6 \end{array} \rightarrow 9y=9 \rightarrow y=1 \rightarrow 2x+7=3 \rightarrow x=-2 \quad x+y=-1 \end{array}$$

- 28. A a = 2b/3 b = 5c/7 so $a = 10c/21 \rightarrow a/c = 10/21$
- 29. C $x^2 8! = 9! \rightarrow x^2 = 9$ so x = +3 3
- 30. A f(5) = 2 f(2) = 1 f(1) = 4 and f(4) = 0