

“NOTA” stands for “None of the Above Answers is correct.”

1. Find the sum of the *distinct* real roots of $f(x) = x^3 - 4x^2 + 5x - 2$.
A. -4 B. 2 C. 3 D. 4 E. NOTA

2. What is the sum of the coefficients in the expansion of $(x+2y)^3(2x-3z)^5$?
A. -27 B. -9 C. 9 D. 27 E. NOTA

3. Find the volume of the parallelepiped spanned by the vectors $\langle 1, 2, 3 \rangle$, $\langle 1, -2, 4 \rangle$, and $\langle 2, -2, 3 \rangle$.
A. -18 B. 18 C. -36 D. 36 E. NOTA

4. The sequence of integers S_0, S_1, S_2, \dots satisfies the recurrence relation $S_n = S_{n-1} + (-1)^n \cdot n^2$, for all integers $n > 0$. If $S_{100} = 5000$, then what is the value of S_{21} .
A. -281 B. 160 C. 181 D. 601 E. NOTA

5. Solve for x : $\sqrt{x + \sqrt{x + \sqrt{x + \dots}}} = 5$.
A. 15 B. 20 C. 25 D. 30 E. NOTA

6. If $a + bi = \frac{a - bi}{a + bi}$, where a and b are real numbers, then what is the value of ab^2 ?
(Note: $i = \sqrt{-1}$)
A. $-\frac{1}{4}$ B. $\frac{3}{4}$ C. $\frac{1}{8}$ D. $-\frac{3}{8}$ E. NOTA

7. Find the product of the values of x for which the matrix $\begin{pmatrix} 1 & -x & x \\ -x & 2 & -x \\ 1 & -x & 3 \end{pmatrix}$ is *not* invertible.
A. -6 B. -3 C. 3 D. 6 E. NOTA

8. How many four-digit positive odd integers can be formed using only the digits 0, 2, 3, 4, and 5, if the leftmost digit cannot be 0 and repetition is allowed?
- A. 200 B. 250 C. 300 D. 500 E. NOTA
9. If $1 + \cos^2 \theta + \cos^4 \theta + \cos^6 \theta + \dots = A$ and $0 < \theta < \pi/2$, then what is the value of $\sin \theta$?
- A. $1/A$ B. $1/\sqrt{A}$ C. \sqrt{A} D. $1/A^2$ E. NOTA
10. Find the coefficient of the constant term in the expansion of $\left(x + \frac{1}{x^2}\right)^{10}$?
- A. $\binom{10}{4}$ B. $\binom{10}{5}$ C. $\binom{10}{7}$ D. $\binom{10}{8}$ E. NOTA
11. The set $\{1, 2, 3, 4, 5, 6\}$ can be ordered in $6!$ different ways. In how many of these orderings does 1 appear before 2 and 2 appear before 3?
- A. 24 B. 60 C. 120 D. 240 E. NOTA
12. What is the maximum value of $f(x) = 3\cos(\pi + x) + 4\sin(\pi + x) - 1$?
- A. 4 B. 5 C. 6 D. 7 E. NOTA
13. If $f(x) = \frac{1+2x}{3x+4}$, then what is the value of $f^{-1}(1) - f^{-1}(-1)$?
- A. -4 B. -2 C. 2 D. 4 E. NOTA
14. Solve for x , where x is a real number: $\frac{10^x}{3+10^x} = 4$.
- A. $\ln(-4)$ B. $-\ln 4$ C. $\frac{1}{\ln 4}$ D. $\ln 4$ E. NOTA
15. If $k = \sin x - \cos x$, then which of the following is equal to $\cos^2(2x)$?
- A. $\sqrt{1-k^2}$ B. $1-k^2$ C. $\sqrt{2k^2-k^4}$ D. $2k^2-k^4$ E. NOTA

16. What is the area of the region bounded by the curves $y = \sqrt{9 - x^2}$ and $y = \frac{2}{3}\sqrt{9 - x^2}$?
- A. $3\pi/2$ B. 3π C. 6π D. 9π E. NOTA
17. If $a = \log 3$, $b = \log 4$, and $c = \log 5$, then which of the expressions is equal to $\log_{\sqrt{3}} 90$?
- A. $\frac{2a+b+c}{a}$ B. $\frac{4a+b+2c}{a}$ C. $\frac{4a+2b+2c}{a}$ D. $\frac{2a+b+c}{2a}$ E. NOTA
18. Compute the limit: $\lim_{x \rightarrow 2} \sqrt{x-2}$.
- A. 0 B. 2 C. 4 D. Doesn't exist E. NOTA
19. If $\sin x = \frac{\sin(3x)}{2}$, then what is the value of $\cos^2 x$?
- A. $\frac{1}{2}$ B. $\frac{3}{2}$ C. $\frac{1}{4}$ D. $\frac{3}{4}$ E. NOTA
20. Find the sum of the three real solutions of $e^{3x} - 8e^{2x} + 18e^x - 10 = 0$.
- A. $\ln 8$ B. e^8 C. $\ln 10$ D. e^{10} E. NOTA
21. Find the distance between the two parallel lines $y = 3x - 4$ and $y = 3x + 4$.
- A. $4\sqrt{10}/5$ B. $\sqrt{10}/8$ C. $4/5$ D. $5/4$ E. NOTA
22. Which of the following is the inverse of the matrix $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$?
- A. $\begin{pmatrix} -1/2 & -1 \\ -3/2 & -2 \end{pmatrix}$ B. $\begin{pmatrix} 4 & -2 \\ -3 & 1 \end{pmatrix}$ C. $\begin{pmatrix} -2 & 1 \\ 3/2 & -1/2 \end{pmatrix}$ D. $\begin{pmatrix} 1 & -2 \\ -3 & 4 \end{pmatrix}$ E. NOTA
23. For how many positive integers $N < 100$ does $\sum_{x=1}^N i^x = 0$? (Note: $i = \sqrt{-1}$)
- A. 23 B. 24 C. 25 D. 26 E. NOTA

24. How many of the positive integer factors of 4200 are multiples of 4?
- A. 6 B. 12 C. 24 D. 48 E. NOTA
25. Find the cosine of the angle between the vectors $\langle 1,1,4 \rangle$ and $\langle 2,3,3 \rangle$.
- A. $\frac{17\sqrt{11}}{66}$ B. $\frac{6\sqrt{11}}{17}$ C. $\frac{17\sqrt{11}}{11}$ D. $\frac{\sqrt{11}}{17}$ E. NOTA
26. Find $x \cdot y \cdot z$ if x, y, z are the solutions to the following system:
- $$\begin{aligned}3yz + xz + xy &= 8xyz \\yz + 2xz + 2xy &= 6xyz \\2yz + 4xz + 3xy &= 13xyz\end{aligned}$$
- A. $-1/6$ B. $1/6$ C. $2/3$ D. -6 E. NOTA
27. What conic section is described by the polar equation $r = \frac{3}{4 + \sin \theta}$?
- A. Parabola B. Hyperbola C. Ellipse D. Circle E. NOTA
28. Find the sum of all the elements of the first 10 rows of Pascal's triangle, if the first row only contains 1.
- A. 511 B. 1023 C. 2046 D. 2047 E. NOTA
29. What is the discriminant of the polynomial $f(x) = 2x^2 - 4x + 10$?
- A. $8i$ B. -64 C. $4\sqrt{6}$ D. 96 E. NOTA
30. How many increasing arithmetic progressions which have 12 terms have each term being a positive integer less than 101?
- A. 340 B. 360 C. 395 D. 405 E. NOTA