

For all questions, answer E. "NOTA" means none of the above answers is correct.

1. Find $f'(2)$ if $f(x) = (x^3 - 2)(3x^2 + 4)(x - 7)$.
A) -1224 B) -1168 C) -1040 D) -932 E) NOTA

2. Determine the interval(s) upon which the function $g(x) = x^3 - x^2 - x + 1$ is concave up.
A) $x \in (\frac{1}{3}, \infty)$ B) $x \in (-\infty, \infty)$ C) $x \in (-\infty, -\frac{1}{3})$ D) $x \in (\frac{1}{3}, 1)$ E) NOTA

3. A rectangle is inscribed in the region bounded by the graphs of $2y = 1 - 2x^2$ and $y = 0$. Find the perimeter of the rectangle of maximum area.
A) $\frac{1}{3}(1 + \sqrt{6})$ B) $\frac{1}{2}(1 + \sqrt{6})$ C) $\frac{2}{3}(1 + \sqrt{6})$ D) $\frac{3}{4}(1 + \sqrt{6})$ E) NOTA

4. Evaluate: $\lim_{x \rightarrow C} \frac{x^2 - C}{x - C}$
A) 0 B) $2C$ C) C D) the limit does not exist E) NOTA

5. Find the total area bounded by the graphs of $x = y^3 - 4y$ and $x = 0$.
A) 2 B) 4 C) 6 D) 8 E) NOTA

6. Find $\frac{dy}{dx}$ if $y = 1 - \cos 2x + 2\cos^2 x$.
A) 0 B) $-\sin 2x + 4 \cos x \sin x$ C) $2 \sin 2x + 4 \cos x$ D) $-\sin 2x - 4 \cos x \sin x$ E) NOTA

7. Determine the value(s) of C that satisfy the conclusion of the Mean Value Theorem for Derivatives given that $h(x) = 2x^3 - 9x - 1$ on the interval $[-4, 2]$.
A) $C = \pm 2$ B) $C = \pm \frac{3}{2}$ C) $C = \pm 1$ D) $C = 0$ E) NOTA

8. Use the trapezoidal rule with $n = 6$ to approximate the area of the region below the curve $y = x^2 + 1$, above the x -axis and between the lines $x = 0$ and $x = 3$.
A) 12.125 B) 12.375 C) 12.625 D) 12.875 E) NOTA

9. Determine the number of points of inflection on the graph of $f(x) = \frac{1}{20}x^5 - \frac{1}{6}x^4 + 5x - 2$.
A) 0 B) 1 C) 2 D) 3 E) NOTA

10. Evaluate $\lim_{x \rightarrow -\infty} \left(\sqrt{x^2 - \frac{4}{3}x} + 2 + x \right)$.

- A) $\frac{4}{3}$ B) 2 C) $\frac{8}{3}$ D) the limit does not exist E) NOTA

11. Given the function $g(x) = (x-1)^2 + 2$, find the sum of slopes of the tangent lines at the following values of x : $x = -8$, $x = -5$, $x = -2$, $x = 1$, $x = 4$, $x = 7$, and $x = 10$.

- A) 0 B) 6 C) -6 D) 12 E) NOTA

12. Evaluate $\int_0^1 e^x dx$.

- A) $\frac{e^6 - 1}{6}$ B) $5e^4$ C) e^5 D) $\frac{e^6 - 1}{6}$ E) NOTA

13. If $xe^y + 2x^2y = y \ln x - 4y$, find $\frac{dy}{dx}$ at the point $(1,0)$.

- A) $-\frac{1}{6}$ B) $-\frac{1}{7}$ C) $\frac{1}{7}$ D) $\frac{1}{6}$ E) NOTA

14. Triangle DCA has vertex D at the point $(-2,0)$, vertex C at the point $(3,0)$, and vertex A on the curve $h(x) = 3x - x^3$, where $-\sqrt{3} < x < \frac{\sqrt{2}}{2}$. Find the maximum possible area of triangle DCA.

- A) $\frac{25\sqrt{2}}{4}$ B) $\frac{5}{4}$ C) $\frac{5}{2}$ D) 5 E) NOTA

15. Evaluate $\int_2^5 \ln\left(\frac{1}{e^{\frac{1}{x}}}\right) dx$.

- A) $\ln(2.5)$ B) $\ln(0.4)$ C) $\ln(3)$ D) $\ln(1.5)$ E) NOTA

16. Find the solution to the differential equation $\frac{dy}{dx} \ln x + \frac{y}{x} = \frac{dy}{dx}$ which passes through the point $(e^2, 3)$.

- A) $y = 3e^x - (3e^{e^2} - 3)$ B) $y = 3\ln x - 3$ C) $y = \frac{3}{\ln x - 1}$ D) $y = \frac{e}{\log x}$ E) NOTA

17. Determine the volume of the solid formed by revolving the region bounded by the graphs of $y = \sqrt{-x^2 + 4x + 5} - 4$ and $y = -4$ about the line $x = 2$.

- A) 9π B) 18π C) 27π D) 36π E) NOTA

18. Use Newton's Method to calculate x_2 in approximating a zero for $f(x) = e^{2x} - 3x^2$ with an initial guess of $x_0 = 0$. The value of x_2 can be written in the form $\frac{-A - Be}{A + Ce}$, where A and B are relatively prime, positive integers. Find $A + B - C$.
- A) -7 B) -1 C) 17 D) 23 E) NOTA
19. An ellipse, which at time $t = 0$ is represented by the equation $64x^2 - 384x + 9y^2 + 36 = 0$, has a major axis whose length is increasing at a rate of 6 units/sec and a minor axis whose length is decreasing at a rate of 1 unit/sec. At what rate, in units²/sec, is the area of the ellipse changing at time $t = 2$ sec?
- A) -6π B) -4π C) $-\pi$ D) 10π E) NOTA
20. To how many of the following does Rolle's Theorem apply?
- I. $f(x) = \sin x$ on $[0, 2010\pi]$
- II. $f(x) = \frac{1}{x}$ on $[-1, 1]$
- III. $f(x) = x^2 + 1$ on $[-2010, 2010]$
- IV. $f(x) = \ln|x|$ on $\left[\frac{1}{2010}, 2010\right]$
- A) 1 B) 2 C) 3 D) 4 E) NOTA
21. Let R be the region bounded by the graphs of $y = x$ and $y = x^4$. Find the positive difference between the volumes of the solids generated by revolving R about the x -axis and by revolving R about the y -axis.
- A) $\frac{\pi}{9}$ B) $\frac{\pi}{15}$ C) $\frac{\pi}{24}$ D) $\frac{\pi}{36}$ E) NOTA
22. Find the sum of all value(s) of C that satisfy the conclusion of the Mean Value Theorem for Integrals given that $g(x) = x^2 + 1$ on $[0, 6]$.
- A) 0 B) $-2\sqrt{3}$ C) $3\sqrt{2}$ D) $-3\sqrt{2}$ E) NOTA
23. Find $\frac{d^2y}{dx^2}$ for the relation $xy + y^2 = 1$.
- A) $\frac{-y}{x+2y}$ B) $\frac{2xy+2y}{(x+2y)^3}$ C) $\frac{2}{(x+2y)^3}$ D) $\frac{2x+2y^2}{(x+2y)^3}$ E) NOTA
24. Evaluate $\int_{\frac{\sqrt{3}}{3}}^1 \frac{\arctan x \, dx}{x(x + \frac{1}{x})}$.
- A) $\frac{5\pi^2}{288}$ B) $\frac{5\pi^2}{192}$ C) $\frac{5\pi^2}{144}$ D) $\frac{5\pi^2}{72}$ E) NOTA

25. Find $f\left(\frac{1}{2010}\right)$ if $f(x) = \log_{2010} \cos^2 2010x + e^{2010x} + \log_{2010} \sec^2 2010x + \pi^{e^\pi}$.
- A) $2010e^{2010}$ B) $2009e$ C) $2011e$ D) $2010e$ E) NOTA
26. A cylindrical oil tank is 8 feet deep and 20 feet in diameter. At $t=0$ minutes, the drain is opened so that the oil will flow out at a rate directly proportional to the square root of the volume of oil in the tank at any time. If the tank was full of oil initially, after how many minutes will it be empty if the oil initially flowed out at $20\pi \text{ ft}^3/\text{min}$?
- A) 48 B) $64\sqrt{2}$ C) 80 D) $84\sqrt{2}$ E) NOTA
27. The base of a solid is bounded by the graphs of $y = x^2$, $x = 4$, and $y = -x^2$. Find the volume of the solid, in cubic units, if the cross-sections taken perpendicular to the x -axis are semicircles (the diameters of the semicircles lie on the base of the solid).
- A) $\frac{256\pi}{5}$ B) $\frac{512\pi}{5}$ C) $\frac{1024\pi}{5}$ D) $\frac{2048\pi}{5}$ E) NOTA
28. What is the average value of the function $f(x) = \frac{x^2 + 3x - 4}{x^3 - 4x^2 + 4x}$ on the interval $[3, 9]$?
- A) $\frac{18}{7} + \ln\left(\sqrt[6]{\frac{49}{3}}\right)$ B) $\ln\left(e^{\frac{108}{7}} \sqrt[6]{\frac{49}{3}}\right)$ C) $\frac{108}{7} - \ln\left(\sqrt[6]{\frac{49}{3}}\right)$ D) $\ln\left(e^{\frac{3}{7}} \sqrt[6]{\frac{49}{3}}\right)$ E) NOTA
29. Find $f''(2)$ given that $f(x) = -\int_{x^2}^4 (3t+2)dt$.
- A) 12 B) 14 C) 56 D) 76 E) NOTA
30. Evaluate $\int \sqrt{2+\sqrt{x}} dx$
- A) $\frac{4}{15}(2+\sqrt{x})^{\frac{3}{2}}(3\sqrt{x}+4)+C$ B) $\frac{4}{15}(2+\sqrt{x})^{\frac{3}{2}}(3\sqrt{x}+2)+C$
C) $\frac{4}{15}(2+\sqrt{x})^{\frac{3}{2}}(3\sqrt{x}-2)+C$ D) $\frac{4}{15}(2+\sqrt{x})^{\frac{3}{2}}(3\sqrt{x}-4)+C$
E) NOTA