Section I – Trivia 101

For this section, determine the answer to each question, then change each letter in the answer to a number by using A=1, B=2, C=3, etc. Add these numbers, then report the sum as your team's answer. For example, if the question is "Who plays Batman in the most recent Batman movies?", the answer is Christian Bale, and the answer your team will submit is 121 (3+8+18+9+19+20+9+1+14+2+1+12+5=121).

1. What band originally sang the song "Pinball Wizard" as part of their rock opera *Tommy*? (2 words, 1 point)

2. What 1980s fantasy action film's tagline was "There can be only one."? (1 word, 2 points)

3. What sitcom features a regional manager whose name is David Brent in the British version, Gilles Triquet in the French version, and Bernd Stromberg in the German version? (2 words, 2 points)

4. What book, written in 1950 by C.S. Lewis, was released as a movie in December 2005? (7 words, 3 points)

5. What magazine published its 1000th issue in 2006 with a cover that was an homage to the Beatles' album *Sgt. Pepper's Lonely Hearts Club Band*? (2 words, 3 points)

6. What fake news organization features editorials from such personalities as Smoove B, a smooth-talking ladies man; Jackie Harvey, a clueless celebrity spotter; Gorzo the Mighty, Emperor of the Universe, a 1930s style sci-fi villain; and Herbert Kornfeld, an accounts receivable supervisor who speaks in ebonics? (2 words, 5 points)

7. What film director made his debut with the movie *Bottle Rocket*, but who is more commonly known as the director of *The Royal Tenenbaums*? (2 words, 5 points)

8. What privately held company distributes "authoritative" consumer versions of "important classic and contemporary films" totaling over 400 DVDs issued, including such classics as *8½* and *Seven Samurai*, and cult classics such as *Brazil* and *Videodrome*? (3 words, 7 points)

9. In my opinion, his best works are *Diary*, *Lullaby*, and *Rant: An Oral Biography of Buster Casey*. I'm not a big fan of *Invisible Monsters*. Who is this author? (2 words, 10 points)

10. My favorite album of all-time (thus far) has to be the 2002 album by The Flaming Lips. What is the name of this album? (5 words, 12 points)

Section II – What Number am I?

For this section, a number of clues will all describe the same positive integer. The answer your team will submit is the integer being described.

1. the length, in yards, of a cricket pitch; Emmitt Smith's jersey number while he played for the Dallas Cowboys; the number in the title of Joseph Heller's famous work; the number of letters in the Hebrew alphabet (2 points)

2. the sixth Catalan number; the smallest integer *n* such that the sum of all two-digit numbers made from the digits of *n*, without repetition, equals *n* (4 points)

3. the highest jersey number allowed in the National Hockey League; a number in one of the versions of Microsoft's operating system Windows; the atomic number of the element Californium (3 points)

4. the maximum number of givens on a Sudoku board that still does not yield a unique solution; the number at the end of the Talking Heads' debut album (5 points)

5. the only number in base 10 whose representations in bases 3 through 8 are two digits long (2 points)

6. the fifth Woodall number; the product of the first and fifteenth odd primes (5 points)

7. the number of counties in California; the number of home runs hit by Ryan Howard for the Philadelphia Phillies in 2006; the sum of the first seven prime numbers (2 points)

8. the magic constant of a normal 4x4 magic square (1 point)

9. a double factorial; the smallest integer *n* such that the factorization of $x^n - 1$ over rational numbers includes coefficients other than 1 and -1; the larger number in the fifth Ruth-Aaron pair; the number halfway between the smallest and largest numbers in the third prime quadruplet (10 points)

10. a prime number; a Fibonacci number; a Markov number; an Eisenstein prime; the sum of its positive integer factors is a two-digit number (16 points)

Section III – Unscramble Me

For this section, each question is a set of at least two related math words or terms with their letters combined and rearranged in alphabetical order. Once your team unscrambles the letters into the component words, there will be an additional word that belongs with the others. The answer your team will submit is the additional word. For example, BEGHIORSTTU would unscramble to RIGHT and OBTUSE, with ACUTE being the additional word (and thus the answer your team would submit). Each question is worth 5 points.

1. AAAACCDDDDDDEEEEEEEHHHHHINNNNOOOOOORRRRRSTTX

2. AAACEEEEGILLLMNNOPRRRTTUX

3. EEEFHNOORRTU

4. AABCDEEILLRSSUU

5. AACCEHHIIIMMNORRTT

6. AADEEIMMNN

7. AAAEHHLPTT

8. AEEEGIIINOPSTTVV

9. AADDEEFGGIIILLMNNOOPRRTU

10. AAABCEEEGGIILMORRSSSTTTTY

Section IV – Somewhat Difficult Problems

For this section, solve the problem. Difficulty ranges from moderately difficult to extremely difficult. Each question is worth 5 points.

1. If three distinct people are playing Rock, Paper, Scissors, in how many ways can they each throw with there being a clear winner? For example, there would not be a clear winner if each of the three made a distinct throw or if all three threw the same thing; or, for example, if two people threw rock and one threw scissors. There would be a clear winner if one person threw paper and two people threw rock.

2. Let $\lfloor x \rfloor$ represent the largest integer n such that $n \le x$. There is no positive solution to the equation $x\lfloor x \rfloor = 2010$. In fact, what is the last year c before 2010 in which there was a positive solution to the equation $x\lfloor x \rfloor = c$?

3. The first term of a geometric sequence is 1 and the last term of the sequence is 16. If the sequence consists only of integers, how many such sequences exist?

4. Let A and B be nonzero rational numbers. If the function $f(x) = Ax^2 + Bx - \frac{16}{27}$ has A and B as its roots, what is the numerical value of B?

5. The sum of the first n terms of a sequence of integers is $n^2 + 1$. How many terms in the sequence are even?

6. A sequence $\{a_n\}$ with $a_1 = 2$ and $a_2 = 6$ is defined in the following way: for each term beyond the second, $a_n =$ the sum of the digits of the sum of the squares of a_{n-1} and a_{n-2}. What is the value of a_{2010}?

7. What is the smallest positive integer n such that the sum of the first n positive integers is divisible by 24?

8. In the equation $\sqrt{a+4\sqrt{5}} + \sqrt{a-4\sqrt{5}} = \sqrt{b+4\sqrt{5}}$, where a and b are positive integers, b must be equal to what number?

9. Let $X = \dfrac{\sum_{i=2010}^{10000} i!}{2010!}$, and let x_k be the remainder when X is divided by k. Find $\sum_{i=2011}^{2016} x_i$.

10. What is the probability that when dealt two cards each from a standard deck of 52 cards, Statler and Waldorf have the same rank on each others' cards? For example, both could have a 10 and a 2 or both could have two jacks.

11. If $\lceil x \rceil$ is the least integer greater than or equal to x, find the sum $\sum_{n=1}^{2010} \left\lceil \sqrt{2n+0.25} - 0.5 \right\rceil$.

12. A Pythagorean triple with consecutive integer leg lengths will have a hypotenuse whose length may end in three of the ten digits. Which three?

13. Let A and B be positive integers. If the functions $f(x) = x^3 - Ax - B$ and $g(x) = x^2 - Ax + B$ have a common integer root, what is the smallest possible odd value of B?

14. A powerful number is defined as any positive integer such that for every prime number p dividing it, p^2 also divides it. How many powerful numbers less than or equal to 1000 exist?

15. n is the smallest positive integer such that $\frac{n}{2}$ equals some integer squared, $\frac{n}{3}$ equals some integer cubed, $\frac{n}{11}$ equals some integer raised to the eleventh power, and $\frac{n}{13}$ equals some integer to the thirteenth power. The prime factorization of n is $p_1^{n_1} p_2^{n_2} \ldots p_k^{n_k}$, where p_1, p_2,…, p_k are distinct primes written in increasing order and n_1, n_2,…, n_k are positive integers. Find the value of $\sum_{i=1}^{k} (p_i - n_i)(-1)^i$.

Section V – Minority Game

For this section, choose one of the two options. For each question, a team will score points if that team chooses the option that is chosen by the fewest teams. For example, if the options are quadratic or cubic, and if 25 teams choose quadratic and 17 teams choose conic, those 17 teams that chose conic will get points for that question. Make sure other teams don't notice what you are selecting for each one. Each question is worth 5 points.

1. (Nintendo Wii) OR (Xbox 360)
2. (Mac) OR (PC)
3. (Facebook) OR (Myspace)
4. (Harry Potter) OR (Twilight)
5. (Pizza) OR (Chicken Fingers)
6. (Jon) OR (Kate)
7. (Jay Leno) OR (Conan O'Brien)
8. (John Locke) OR (Jack Shephard)
9. (iPhone) OR (Sidekick)
10. (The Hills) OR (The City)

Section VI – Game Theory

For this section, describe the optimal strategy one should employ in each situation in order to achieve the desired end result. Assume that all participants will behave in a manner that maximizes his or her benefit, not necessarily how people would actually play it haphazardly. Each question is worth 5 points.

1. There are five pirates who find 100 gold coins and must decide how to distribute them. The pirates have a strict order of seniority, i.e., there is a head pirate, a second in command, etc. The most senior pirate will propose a distribution of coins, and all of the pirates will vote on whether to accept this distribution, with the proposing pirate casting the deciding vote in the event of a tie. If the proposal is approved by vote, the coins are distributed in that manner; if not, the proposing pirate is thrown overboard, and the next most senior pirate makes a new proposal, beginning the game again. What

distribution should the head pirate propose in order to be given the most coins possible and in order not to be thrown overboard?

2. Two suspects are arrested by the police. The police have insufficient evidence for a conviction, and, having separated both prisoners, visit each of them to offer the same deal. If one testifies (defects from the other) for the prosecution against the other and the other remains silent (cooperates with the other), the betrayer goes free and the silent accomplice receives the full 10-year sentence. If both remain silent, both prisoners are sentenced to only six months in jail for a minor charge. If each betrays the other, each receives a five-year sentence. Each prisoner must choose to betray the other or to remain silent. Each one is assured that the other would not know about the betrayal before the end of the investigation. How should each prisoner act in order to minimize his or her time in prison?

3. Several people are to guess a real number between 0 and 100. The number they are trying to guess is what 2/3 of the average of all guesses would be, with the winner being the one whose guess is closest to 2/3 of the average of all guesses. If you are one of these people, what number should you guess?

4. An auctioneer is to auction off a standard dollar bill between two people with the following rule: the dollar goes to the higher bidder, who pays the amount he bids. The lower bidder also must pay the highest amount that he bid, but gets nothing in return. If both bidders are trying to maximize their profit, what will happen with the bidding for this dollar bill?

5. Two people have a sum of money, say 100 $1 bills. The first person decides how to divide the money (in $1 increments) between the two people, and the second person either accepts or rejects this proposal. If the second player rejects the proposal, neither player receives anything. If the second player accepts, the money is split according to the proposal. What proposal should the first person suggest in order to maximize the amount of money she receives?

Section VII – Sum of Four Perfect Squares

Lagrange's four-square theorem states that any natural number can be represented as the sum of no more than four positive perfect squares. For this section, determine a set of positive perfect squares whose sum is the number. For example, if the number was 65, 65 can be written as the following sums: 1+64, 16+49, 4+25+36, or possibly in another way. Any of these representations will be acceptable. Each question is worth 1 point. See answer sheet for natural numbers representing the sums.

Section VIII – The 2010 Game

For this section, your team will attempt to write each of the first 50 positive integers using only the digits 2, 0, 1, and 0 (you must use all four digits in the representation), along with the symbols +, -, *, /, !, and $\sqrt{\ }$, along with exponentiation and juxtaposition, and symbols for grouping. Each question is worth 1 point.

Section IX – Cryptograms

For this section, your team will determine the mathematical quotation and its author that are coded in the cryptogram. Each question is worth 10 points .

1. AFTET DX PNAFDPH XAESPHT DP AFT ODEOKT ITDPH AFT NEDHDP NZ SPU SPL TCTEU WSECTK.
   -SEDXANAKT

2. R FGE'A PLTRLUL RE NDACLNDARIO. -DTPLWA LREOALRE

3. HBIDXHBIMOP MP B SBHX CZBQXV BOOEUVMTS IE OXUIBMT UYZXP AMID HXBTMTSZXPP HBULP ET CBCXU. -VBJMV DMZGXUI

4. M AMNOTAMNFPFME FL M RUFEC AME FE M CMXS XZZA UZZSFEH KZX M RUMPS PMN IOFPO FLE'N NOTXT. -POMXUTL CMXIFE

5. TRUCKTRUXNY XY UCK APFS YNXKPNK ECKDK APK PKQKD VPAEY ECRU APK XY URFVXPM RLAJU PAD ECKUCKD ECRU XY YRXI XY UDJK. -LKDUDRPI DJYYKFF

Section X – Wild Card Question

For this section, each team will answer with a positive integer. The team that answers with the smallest unique integer will earn that number of points for this question while all other teams will earn 0 points. For example, if the numbers that are written down for this problem are 1, 1, 3, 3, 3, 4, 7, 7, 8, 10, 12, and 15, then the team that answered 4 will earn 4 points and all other teams will earn 0 points. Make sure other teams don't notice what your team puts down as your answer.