QUESTION 0

Theta

Solve for x: $4x-5=23$

$4x=28$

$x=7$

$\boxed{A=7}$

Alpha

Evaluate $\frac{A!}{5!}$

$\frac{7!}{5!}=\frac{7(6)(5!)}{5!}=7(6)=42$

$\boxed{B=42}$

Calculus

Find $\int_1^2 Bx\,dx$

$\int_1^2 42x\,dx=21x^2\Big|_1^2=84-21=63$

$\boxed{C=63}$

QUESTION 1

Theta

Let $f(x)=4x^2+7x+5$. If S is the sum of the roots of $f(x)$, and P is the product of the roots of $f(x)$, find the value of $(S+P)^2$.

$S=\frac{-7}{4}$ and $P=\frac{5}{4}$, so $(S+P)^2=\left(\frac{-7}{4}+\frac{5}{4}\right)^2=\frac{1}{4}$

$\boxed{A=\frac{1}{4}}$

Alpha

Let $f(x)=\frac{1}{A}\cos(A\pi x)+2010$. If M is the amplitude of the graph of $f(x)$, and P is the period of the graph of $f(x)$, then find the value of MP.

$M=1\div\frac{1}{4}=4$, and $P=\frac{2\pi}{\frac{1}{4}\pi}=8$, so $MP=4(8)=32$.

$\boxed{B=32}$

Calculus

What is the volume of the resulting solid formed when the region bounded by $y = x^B$, $x = 1$ and $y = 0$ is revolved about the $x-$axis?

$\pi\int_0^1 \left(x^{32}\right)^2 \ dx = \pi\int_0^1 \left(x^{64}\right) \ dx = \frac{\pi}{65}x^{65}\Big|_0^1 = \frac{\pi}{65}$.

$\boxed{C = \frac{\pi}{65}}$

QUESTION 2

Theta

What is the value of $r + s + tu$ if

$\begin{bmatrix} -1 & 4 \\ 2 & 5 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ 3 & 7 \end{bmatrix} \cdot \begin{bmatrix} 8 & -5 \\ -4 & -10 \end{bmatrix} = \begin{bmatrix} r & s \\ t & u \end{bmatrix}$?

$\begin{bmatrix} -1 & 4 \\ 2 & 5 \end{bmatrix} + \begin{bmatrix} 16-4 & -10-10 \\ 24-28 & -15-70 \end{bmatrix} = \begin{bmatrix} 11 & -16 \\ -2 & -80 \end{bmatrix}$, so $r + s + tu = 11 - 16 - 2(-80) = 155$.

$\boxed{A = 155}$

Alpha

Consider the digits of A as the form of a number that is ALREADY in base-9. When A is changed to a base-10 number, what is the largest digit of the base-10 number?

$1(81) + 5(9) + 5(1) = 81 + 45 + 5 = 131_{10}$; largest digit is 3.

$\boxed{B = 3}$

Calculus

A sphere's radius is increasing at a rate of B feet per second. What is the instantaneous rate of change of its volume, in cubic feet per second, at the instant that the radius is 5 feet?

$V = \frac{4}{3}\pi r^3$, so $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} = 4\pi(5)^2(3) = 300\pi$.

$\boxed{C = 300\pi}$

QUESTION 3

Theta

What is the length of the major axis of the ellipse with equation $x^2 + 2y^2 + 6x + 8y - 4 = 0$?

$(x+3)^2 + 2(y+2)^2 = 4 + 9 + 8 = 21$, so $\frac{(x+3)^2}{21} + \frac{(y+2)^2}{21/2} = 1$. Major axis is $2a = 2\sqrt{21}$.

$\boxed{A = 2\sqrt{21}}$

Alpha

In ΔPQR, $m\angle Q = 90^\circ$, $PQ = \sqrt{35}$, and $PR = A$. What is the value of $\tan P$?

$\sqrt{35}^2 + (QR)^2 = \left(2\sqrt{21}\right)^2$, so $35 + QR^2 = 84$, and $QR = 7$. $\tan P = \frac{QR}{QP} = \frac{7}{\sqrt{35}} = \frac{7\sqrt{35}}{35}$.

$\boxed{B = \frac{\sqrt{35}}{5}}$

Calculus Find $f'(32)$ if $f(x) = x^2 + x^{\left(\frac{B}{\sqrt{35}} + \frac{6}{5}\right)}$.

$f(x) = x^2 + x^{7/5}$. $f'(x) = 2x + \frac{7}{5}x^{2/5}$. $f'(32) = 2(32) + \frac{7}{5}(32)^{2/5} = 64 + \frac{7}{5}(4) = \frac{320+28}{5} = \frac{348}{5}$.

$\boxed{C = \frac{348}{5} \text{ or } 69\frac{3}{5}}$

QUESTION 4

Calculus

If $x^2 + xy + \frac{3}{2}y^2 = 9$, then find the value of $\frac{dy}{dx}$ at the point $(1,2)$.

$2x + xy' + y + 3yy' = 0$. Substitute $(1,2)$ gives $2 + y' + 2 + 6y' = 0$, and $y' = \frac{-4}{7}$.

$\boxed{A = \frac{-4}{7}}$

Theta

What is the sum of the series $A + A^2 + A^3 + A^4 + \ldots$?

$S = \frac{a_1}{1-r} = \frac{-4/7}{1-(-4/7)} = \frac{-4}{7}\left(\frac{7}{11}\right) = \frac{-4}{11}$.

$\boxed{B = \frac{-4}{11}}$

Alpha

What quadratic polynomial function, in the form $f(x) = x^2 + px + q$, has leading coefficient 1 and exactly two roots which are 9 and $22B$?

Roots are 9 and $22\left(\frac{-4}{11}\right) = -8$, so factors $(x-9)(x+8) = x^2 - x - 72$.

$\boxed{C = \quad f(x) = x^2 - x - 72}$

QUESTION 5

Calculus

If $\int_{2}^{8} \ln x \, dx$ is approximated using a right-hand Riemann sum with 3 subintervals of equal length, and the approximation is equal to $\ln P^2$, what is the value of P?

$\int \frac{1}{x} \, dx = \ln x + C$, so approximation is $2\left(\ln 4 + \ln 6 + \ln 8\right) = 2\ln(4 \cdot 6 \cdot 8) = 2\ln 192 = \ln 192^2$.

$\boxed{A = 192}$

Theta

A special deck of A cards has 12 jokers, and the remaining cards are split evenly among 3 different suits. If 2 cards are drawn at random without replacement, what is the probability that the second card is of the same suit as the first card? Note: Jokers are not part of any suit.
180 cards left after jokers, so 60 cards of each suit. After one card drawn from a suit, 59 cards are left in that suit out of 191 total cards.

$\boxed{B = \frac{59}{191}}$

Alpha

If the probability of an event occurring is B, what are the odds it does not occur?

Odds it does occur is $\frac{59}{191-59} = \frac{59}{132}$, so odds it does not occur is $\frac{132}{59}$.

$\boxed{C = \frac{132}{59}}$

QUESTION 6

Calculus

If $\frac{dy}{dx} = \frac{2x}{y}$, and $x = 1$ when $y = 0$, then the particular solution is $y^2 = P(x)$, where $P(x)$ is what second-degree polynomial?

$\int y \, dy = \int 2x \, dx$, so $\frac{y^2}{2} = x^2 + C$, meaning $0 = 1 + C$, so $C = -1$. $\frac{y^2}{2} = x^2 - 1$, so $y^2 = 2x^2 - 2$.

$\boxed{A = 2x^2 - 2}$

Theta

Given $f(x) = A$. If $(0, w)$ is the vertex, and $(\pm p, 0)$ are the $x-$intercepts (where $p > 0$), what is the value of $p - w$?

Vertex is $(0, -2)$. $2x^2 - 2 = 2(x+1)(x-1)$, so $x-$intercepts are $(\pm 1, 0)$. $p - w = 1 - (-2) = 3$.

$\boxed{B = 3}$

Alpha

If $\cos\phi = \frac{3}{5}$, $0<\phi<\frac{\pi}{2}$, and $\sin\gamma = \frac{B}{7}$, $0<\gamma<\frac{\pi}{2}$, what is the value of $\sin(\phi+\gamma)$?

$\sin(\phi+\gamma) = \sin\phi\cos\gamma + \cos\phi\sin\gamma = \frac{\sqrt{40}}{7}\cdot\frac{4}{5}+\frac{3}{5}\cdot\frac{3}{7} = \frac{8\sqrt{10}+9}{35}$.

$\boxed{C = \frac{8\sqrt{10}+9}{35}}$

QUESTION 7

Alpha

What is the area of a triangle with two sides of length 8 and 10 and the included angle of 60°?

Area$=\frac{1}{2}ab\sin C = \frac{1}{2}(8)(10)\sin 60^\circ = 40\cdot\frac{\sqrt{3}}{2} = 20\sqrt{3}$.

$\boxed{A = 20\sqrt{3}}$

Calculus

The region bounded by $y=0$, $y=x^{1/2}$, $x=\sqrt{2}$, and $x=A$ is the base of a solid. Each cross section perpendicular to the $x-$axis is a square. What is the volume of the solid?

$x^{1/2}$ is the side of each square, so the area of each square is x. Volume is

$$\int_{\sqrt{2}}^{20\sqrt{3}} x\,dx = \left.\frac{x^2}{2}\right|_{\sqrt{2}}^{20\sqrt{3}} = 600-1=599.$$

$\boxed{B = 599}$

Theta

B_{10} is what base-5 number?

$599 = 500+75+20+4 = 4(5^3)+3(5^2)+4(5^1)+4(5^0) = 4344_5$.

$\boxed{C = 4344 \text{ or } 4344_5}$

QUESTION 8

Alpha

What is the second-smallest positive solution to $\tan 6x = 1$?

$6x = \frac{\pi}{4}+\pi k$ for integer k, so $x = \frac{\pi}{24}+\frac{\pi}{6}k$. Positive solutions for $k=0,1,2,\ldots$, so second-smallest solution is $x = \frac{\pi}{24}+\frac{\pi}{6}(1) = \frac{5\pi}{24}$.

$\boxed{A = \frac{5\pi}{24}}$

Calculus

If $y = \sin x \tan x$, find $y'(6A)$.

$y' = \sin x \sec^2 x + \tan x \cos x$. $6A = 6\left(\frac{5\pi}{24}\right) = \frac{5\pi}{4}$. $y'\left(\frac{5\pi}{4}\right) = \frac{-1}{\sqrt{2}}\left(\sqrt{2}\right)^2 + 1\left(\frac{-1}{\sqrt{2}}\right) = \frac{-2-1}{\sqrt{2}} = \frac{-3\sqrt{2}}{2}$.

$\boxed{B = \frac{-3\sqrt{2}}{2}}$

Theta

Simplify $\dfrac{8}{\frac{3}{2}+\frac{B}{3}}$.

$$\frac{8}{\frac{3}{2}-\frac{\sqrt{2}}{2}} = \frac{16}{3-\sqrt{2}}\left(\frac{3+\sqrt{2}}{3+\sqrt{2}}\right) = \frac{48+16\sqrt{2}}{7}$$

$\boxed{C = \frac{48+16\sqrt{2}}{7}}$

QUESTION 9

Alpha

The hyperbola with equation $x^2 - y^2 - 8x + 10y - 58 = 0$ has asymptotes with $y-$intercepts $(0,a)$ and $(0,b)$, where $a > b$. What is the value of a?

$(x-4)^2 - (y-5)^2 = 58 + 16 - 25 = 49$, so $\frac{(x-4)^2}{49} - \frac{(y-5)^2}{49} = 1$. Center is $(4,5)$, and slopes of asymptotes are $\pm\frac{49}{49} = \pm 1$. Larger $y-$intercept will be on the asymptote with slope -1, so $\frac{y_2 - y_1}{x_2 - x_1} = \frac{5-a}{4-0} = -1$, so $a = 9$.

$\boxed{A = 9}$

Calculus

Evaluate $\lim\limits_{x \to A} \frac{x^2 - 100}{x - 10}$

$\lim\limits_{x \to 9} \frac{x^2 - 100}{x - 10} = \frac{81-100}{9-10} = 19$.

$\boxed{B = 19}$

Theta

If $\sqrt{B+\sqrt{B+\sqrt{B+\ldots}}}=\frac{1+\sqrt{Q}}{2}$, then what is the value of Q?

$\sqrt{19+\sqrt{19+\sqrt{19+\ldots}}}=x$, so $x=\sqrt{19+x}$. This means $x^2-x-19=0$, and $x=\frac{1\pm\sqrt{1^2-4(1)(-19)}}{2(1)}=\frac{1\pm\sqrt{77}}{2}$. Since $x>0$, drop the negative answer, so $x=\frac{1+\sqrt{77}}{2}$, and $Q=77$.

$\boxed{C=77}$