

The following were changed at the resolution center at the convention: #7 thrown out, #11 33

1.  $\boxed{\frac{-1}{9}}$  Part A: As  $x \rightarrow 1$ , we can assume that  $x > 0$ , so  $\lim_{x \rightarrow 1} \frac{|x| - x}{x - 1} = 0$ . Part B:
- $$\lim_{x \rightarrow 0} \frac{\cos x - 1}{x} = \lim_{x \rightarrow 0} \frac{(\cos x - 1)(\cos x + 1)}{x(\cos x + 1)} = \lim_{x \rightarrow 0} \frac{-\sin x \cdot \sin x}{x(1 + \cos x)} = \lim_{x \rightarrow 0} \left( -\sin x \cdot \frac{\sin x}{x} \cdot \frac{1}{\cos x + 1} \right) =$$
- $$0 \cdot 1 \cdot \frac{1}{2} = 0. \text{ Part C: } \lim_{x \rightarrow 3} \frac{(1/x) - (1/3)}{x - 3} = \lim_{x \rightarrow 3} \left( \frac{3 - x}{3x} \cdot \frac{1}{x - 3} \right) = \lim_{x \rightarrow 3} \frac{-1}{3x} = \frac{-1}{9}. \text{ Sum the parts.}$$

2.  $\boxed{4.1}$  Part A:  $(f \cdot g)'(3) = f \cdot g' + g \cdot f' = (5)(.7) + (-4)(1.1) = -0.9$
- Part B:  $(g/f)'(-2) = \frac{f \cdot g' - g \cdot f'}{f^2} = \frac{(1)(5) - 0}{1^2} = 5$ . Sum the parts.

3.  $\boxed{-1 \frac{23}{24} \text{ or } \frac{-47}{24}}$  Part A:  $f'(x) = 3x^2 - 2x - 1 = 0 @ x = 1$  and  $\frac{-1}{3}$ . Finding the y-values for these and also for the endpoints shows that  $x = -2$  yields the minimum y-value. Part B: This problem requires the relative extrema of the third derivative, so we have to find the 4<sup>th</sup> derivative  $\Rightarrow y^{(iv)} = 24x - 1 = 0 @ x = \frac{1}{24}$ . There are no values where the derivative does not exist. Sum the parts.

4.  $\boxed{\frac{5\pi^2}{2}}$  Part A: (Disk)  $V = \int_0^\pi \pi \sin^2 x dx = \frac{\pi^2}{2}$ . Part B: (Shell)  $V = \int_0^\pi 2\pi x \sin x dx = 2\pi^2$ . Sum the parts.

5. DCAB

6.  $\boxed{\frac{1}{3}}$  Part A: Not moving means  $v = 0 \Rightarrow v = 6t - 2 = 0 @ x = \frac{1}{3}$ . Part B: Changing direction means  $v$  changes sign  $\Rightarrow v = 3(2t - 3)^2(2)$  and is positive for all values of  $t$ . So, the particle never changes direction, or zero times. Sum the parts.

7.  $\boxed{4}$  Part A diverges by the  $n^{\text{th}}$  term test, so subtract 5. Part B converges by the integral test, so add 3. Part C converges by the comparison test with a geometric series, so add 3. Part D converges by the Ratio Test, so add 3. Sum the parts.

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8.  $\frac{8+75\sqrt{5}}{50}$  Part A:  $\tan \theta = \frac{h}{40} \Rightarrow \frac{d\theta}{dt} = \frac{1}{40} \cos^2 \theta \frac{dh}{dt} \Big|_{h=30} = \frac{1}{40} \left(\frac{4}{5}\right)^2 (10) = \frac{4}{25}$ . Part B:

We need to find  $\frac{ds}{dt}$  when  $x=1, y=2$  and  $\frac{dx}{dt} = \frac{3}{2}$ .

So,  $s^2 = x^2 + y^2 = x^2 + (x^2 + 1)^2 \Rightarrow \frac{ds}{dt} = \frac{3x + 2x^3}{s} \cdot \frac{dx}{dt} \Big|_{x=1} = \frac{3\sqrt{5}}{2}$ . Sum the parts.

9.  $\boxed{\text{I}}$  Part A: Change (so we can use L'Hopital's) to  $\lim_{x \rightarrow \infty} \frac{\sqrt{x}}{e^x} = \lim_{x \rightarrow \infty} \frac{(1/2\sqrt{x})}{e^x} = \lim_{x \rightarrow \infty} \frac{1}{2e^x \sqrt{x}} = 0$ .

Part B: Let

$y = \lim_{x \rightarrow \infty} (x-1)^{\frac{1}{x}} \Rightarrow \ln y = \lim_{x \rightarrow \infty} \left[ \frac{1}{x} \ln(x-1) \right] = \lim_{x \rightarrow \infty} \frac{(1/x-1)}{1}$  (by L'Hopital's). Therefore,

$\ln y = \lim_{x \rightarrow \infty} \frac{1}{x-1} = 0$  and  $y = e^0 = 1$ . Sum the parts.

10.  $\frac{1}{24}$  Part A:

$f(x) = \frac{e^x - e^{\frac{x}{2}}}{2} \Rightarrow f'(x) = \frac{1}{4} \left( 2e^x - e^{\frac{x}{2}} \right) \Rightarrow f''(x) = \frac{1}{8} \left( 4e^x - e^{\frac{x}{2}} \right) \Big|_{x=0} = \frac{3}{8}$ . Part B:

$\frac{dx}{dt} = 3t^2$  &  $\frac{dy}{dt} = 2t \Rightarrow \frac{dy}{dx} = \frac{2}{3t} \Rightarrow \frac{d^2y}{dx^2} = \frac{(3t) \cdot 0 - 2 \cdot \left( 3 \cdot \frac{dt}{dx} \right)}{6t^2} = \frac{(-6/3t^2)}{6t^2} = \frac{-1}{3t^4} \Big|_{t=1} = \frac{-1}{3}$ .

Sum the parts.

11.  $\boxed{31}$  Part A: The rectangle areas sum to  $[2 \cdot f(1) + 2 \cdot f(2) + 2 \cdot f(3)] = 2 + 4 + 20 = 26$ .

Part B: Only 2 rectangles can be inscribed, the others have a height of zero. So the sum of the areas of those 2 rectangles is  $[1 \cdot f(1) + 1 \cdot f(2)] = 4 + 1 = 5$ . Sum the parts.

12.  $\boxed{5}$  Part A: Set  $x'(t) = -4 \cos t = 0 \Rightarrow t = \frac{\pi}{2}, \frac{3\pi}{2} \Rightarrow x = -1, 7$ . Part B: Use implicit

differentiation to get  $\frac{dy}{dx} = \frac{2x+1}{2y}$ . The tangent is vertical when  $y=0$ , hence  $x = -2, 1$ .

Sum the parts.

13.  $\boxed{\text{III only}}$  Statements I and II are both always true by the Extreme Value Theorem. Statement III is false. Consider  $y = x^2$  on  $[1, 4]$ . Then  $f'(c) \neq 0$  for  $1 < c < 4$ .

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14. **10** Part A: The 3<sup>rd</sup> derivative at  $x = 3$  is found in the 4<sup>th</sup> term. It is the coefficient of  $\frac{(x-3)^3}{3!}$ .

So,  $f'''(3) = 7$ . Part B: the 4<sup>th</sup> derivative is part of the coefficient of the 4<sup>th</sup> degree term.

$$\frac{f^{(4)}(0)}{4!} x^4 = \frac{x^4}{(2)(4)} \Rightarrow f^{(4)}(0) = \frac{4!}{8} = 3. \text{ Sum the parts.}$$

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|     | Mu Bowl Answers                       |
|-----|---------------------------------------|
| 1.  | $-\frac{1}{9}$                        |
| 2.  | 4.1                                   |
| 3.  | $-\frac{47}{24}$ or $-1\frac{23}{24}$ |
| 4.  | $\frac{5\pi^2}{2}$                    |
| 5.  | DCAB                                  |
| 6.  | $\frac{1}{3}$                         |
| 7.  | 4                                     |
|     |                                       |
| 8.  | $\frac{8+75\sqrt{5}}{50}$             |
| 9.  | 1                                     |
| 10. | $\frac{1}{24}$                        |
| 11. | 31                                    |
| 12. | 5                                     |
| 13. | III only                              |
| 14. | 10                                    |