Mu School Bowl Question #1 2010 MAΘ National Convention

Let \( A = \lim_{x \to 1} \frac{|x| - x}{x - 1} \)  
Let \( B = \lim_{x \to 0} \frac{\cos x - 1}{x} \)  
Let \( C = \lim_{x \to 3} \frac{1/x - (1/3)}{x - 3} \)

Find the value of \( A + B + C \).

Mu School Bowl Question #2 2010 MAΘ National Convention

If \( f(3) = 5, f'(3) = 1.1, g(3) = -4 \text{ and } g'(3) = 0.7 \), then let \( A = (f \cdot g)'(3) \)

If \( f(-2) = 1, f'(-2) = -5, g(-2) = 0 \text{ and } g'(-2) = 5 \), then let \( B = (g / f)'(-2) \)

Find the value of \( A + B \).

Mu School Bowl Question #3 2010 MAΘ National Convention

Let \( A = \) the \( x \)-coordinate of the absolute minimum of \( f(x) \) if \( f(x) = x^3 - x^2 - x + 1 \) on \([-2, 2]\).

Let \( B = \) the \( x \)-coordinate(s) of the relative extrema of \( f'''(x) \) if \( f(x) = \frac{1}{5}x^5 - \frac{1}{24}x^4 \).

Find the value of \( A + B \).

Mu School Bowl Question #4 2010 MAΘ National Convention

Let \( R \) = the region bounded by \( y = \sin x, x = 0, x = \pi, \text{ and } y = 0 \).

\( A = \) the volume of the solid found by revolving the region \( R \) about the \( x \)-axis.

\( B = \) the volume of the solid by revolving the region \( R \) about the \( y \)-axis.

Find the value of \( A + B \).
Using the graph of \( f(x) \) below, list the following quantities from smallest to largest using the letters as designated. The graph is periodic with a period of 1 and the maximum value is at \((1, 1)\).

\[
A = \int_{0}^{2} f(x) \, dx \quad B = \int_{0}^{2} \sqrt{f(x)} \, dx \quad C = \int_{0}^{2} f^2(x) \, dx \quad D = \int_{0}^{2} f'(x) \, dx
\]

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The position of a particle moving along a horizontal line at any time \( t \) is given by \( s(t) = 3t^2 - 2t - 8 \). Let \( A \) = the value(s) of \( t \) for which the particle is NOT moving.

The position of a particle moving along a horizontal line at any time \( t \) is given by \( s(t) = (2t - 3)^3 \). Let \( B \) = the number of times that the particle changes direction.

Find the value of \( A + B \).

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Start with \( S = 0 \) and for each series below, add 3 to \( S \) if the series converges and subtract 5 from \( S \) if the series diverges. Using this rule, what is the value of \( S \)?

\[
\sum_{n=1}^{\infty} \frac{5^n}{3n^2 + 1} \quad \sum_{n=1}^{\infty} \frac{1}{n(\ln n)^2} \quad \sum_{n=1}^{\infty} \frac{\sin n}{3^n} \quad \sum_{n=1}^{\infty} \frac{n \cdot 3^n}{n!}
\]
A balloon rises straight up at 10 ft/sec. An observer is 40 ft. away from the spot where the balloon left the ground. Let $A =$ the rate of change (in radians/sec) of the balloon’s angle of elevation when the balloon is 30 ft. off the ground.

A point moves along the curve $y = x^2 + 1$ such that its $x$–coordinate is increasing at the rate of 1.5 units/sec. Let $B =$ the rate (in units/sec) at which the point’s distance from the origin is changing when the point is at $(1, 2)$.

Find the value of $A + B$. (ignore all units when summing)

Let $A = \lim_{x \to \infty} (e^{-x} \sqrt{x})$  
Let $B = \lim_{x \to \infty} (x - 1)^{\frac{1}{x}}$

Find the value of $A + B$.

If $f(x) = \frac{e^x - e^2}{2}$, then let $A = f''(0)$.

If $x = t^3 - 1$ and $y = t^2$, then let $B = \frac{d^2y}{dx^2}$ at $t = 1$.

Find the value of $A + B$.

Let $A = \text{the approximation of } \int_{0}^{6} (x^2 - 2x + 2) dx \text{ using a lower Riemann sum with 3 inscribed rectangles of equal width on the } x - \text{axis}$.

Let $B = \text{the approximation of } \int_{0}^{4} (x^2 - 6x + 9) dx \text{ using 4 inscribed rectangles of equal width on the } x - \text{axis}$.

Find the value of $A + B$.
Mu School Bowl             Question #12  2010 MAΘ National Convention

A curve is given parametrically by the equations \( x = 3 - 4\sin t \) and \( y = 4 + 3\cos t \) for \( 0 \leq t \leq 2\pi \).

Let \( A = \) all \( x \)-coordinates at which the curve has a vertical tangent.

Let \( B = \) all \( x \)-coordinates on the curve \( x^2 - y^2 + x = 2 \) where the tangent line is vertical.

Find the value of \( A + B \).

Mu School Bowl             Question #13  2010 MAΘ National Convention

If \( f \) is a continuous function on the interval \([a, b]\), which of the following statement(s) is/are NOT necessarily true?

I. \( f \) has a minimum on \([a, b]\)
II. \( f \) has a maximum on \([a, b]\)
III. \( f'(c) = 0 \) for some number \( c, a < c < b \)

Mu School Bowl             Question #14  2010 MAΘ National Convention

The Taylor Series of a function about \( x = 3 \) is given by

\[
f(x) = 1 + \frac{3(x-3)}{1!} + \frac{5(x-3)^2}{2!} + \frac{7(x-3)^3}{3!} + \cdots + \frac{(2n+1)(x-3)^n}{n!} + \cdots
\]

Let \( A = \) the value of \( f''''(3) \), the 3rd derivative of \( f \) at \( x = 3 \).

The Maclaurin Series for a function is given by \( \sum_{n=0}^{\infty} \frac{x^n}{2n} \).

Let \( B = \) the value of \( f^{(4)}(0) \), the 4th derivative of \( f \) at \( x = 0 \).

Find the value of \( A + B \).