1. $\boxed{15\frac{5}{6} \text{ or } \frac{95}{6}}$ Part a: $3^5 = 243$, so $a = 5$. Part b: $4\sqrt[3]{2} = 2^2 \cdot 2^{1/3} = 2^{7/3}$, so $b = \frac{7}{3}$. Part c: $7^{-3} = \frac{1}{343}$, so $c = -3$. Part d: $\left(\frac{1}{2}\right)^3 = \frac{1}{8}$, so $d = 3$. Part e: $9^x = 27 \Rightarrow x \text{ (and } e) = \frac{3}{2}$. Part f: $\left(\sqrt{5}\right)^x = 125\sqrt{5} \Rightarrow 5^{x/2} = 5^{7/2}$, so $x$ (and $f$) $= 7$. Sum the parts.

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2. $\boxed{-9\frac{1}{6} \text{ or } \frac{-55}{6}}$ Part A: The value of z so that $2z + 3 = -9 \Rightarrow z = -6$. So, $h(2 \cdot -6 + 3) = \frac{-17}{2}$. Part B: $h^{-1}(4) =$ the value of $x$ so that $h(x) = 4$ or $\frac{16-2x}{5+x} = 4$ & $x = \frac{-2}{3}$. Sum the parts

parts

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3. $\boxed{\frac{9+\sqrt{15}}{2} \text{ or } 4.5 + \frac{\sqrt{15}}{2}}$ Part A: The denominator is 1 greater than the whole given expression. So, we rewrite the problem as

$$x = 1 + \cfrac{8}{2 + \cfrac{8}{2 + \cfrac{8}{2 + \cfrac{8}{2 + \ldots}}}} \Rightarrow x = 1 + \frac{8}{x+1}.$$ Solving this gives $x = 3$.

Part B: The 1st denom. is not the same as our expression, but the 2nd one is. So, we rewrite it as

$$x = 3 + \cfrac{1}{2 + \cfrac{1}{3 + \cfrac{1}{2 + \ldots}}} = 3 + \cfrac{1}{2 + \cfrac{1}{x}}.$$ This simplifies to a quadratic and using the formula,

we get $x = \frac{3+\sqrt{15}}{2}$. The answer must be positive. Sum the parts.

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4. $\boxed{21\frac{1}{2} \text{ or } \frac{43}{2}}$ Part A: Squaring both sides, we get $4x + 1 - 2\sqrt{(4x+1)(x-11)} + x - 11 = 2x - 4$ or $3x - 6 = 2\sqrt{4x^2 - 43x - 11}$. Squaring again gives: $9x^2 - 36x + 36 = 16x^2 - 172x - 44 \Rightarrow 7x^2 - 136x - 80 = 0$. This factors to: $(x-20)(7x+4) = 0$. The only solution that works is $x = 20$. Part B: Cubing both sides, we get $x^3 - x^2 - 10 = (x-1)^3 = x^3 - 3x^2 + 3x - 1 \Rightarrow 2x^2 - 3x - 9 = 0$. Factoring gives the solutions $x = 3$ and $x = \frac{-3}{2}$. Sum all the solutions.

5. $\boxed{1133}$ Part A: All of the parentheses subtract to 2. But, how many? So, we rewrite the problem:
$1000+(-998+998)+(-996+996)+(-994+994)+...+(-334+334)-332$
$=1000-332=668$. Part B: All of the numbers are perfect squares. Rewrite as
$30^2-29^2+28^2-27^2+...+4^2-3^2+2^2-1^2=$
$(30^2-29^2)+(28^2-27^2)+...+(4^2-3^2)+(2^2-1^2)=59+55+...+7+3.$
This is an arithmetic series and the sum is 465. Sum the parts to get the final answer.

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6. $\boxed{65{,}520}$ M: This simplifies to $3^{2^{2^4}}=3^{2^{16}}=3^{65536}$. D: This simplifies to
$\left(\left(\left((3)^2\right)^2\right)^2\right)^2=\left(\left((3)^4\right)^2\right)^2=\left((3)^8\right)^2=3^{16}$. So, $\frac{M}{D}=\frac{3^{65536}}{3^{16}}=3^{65520}$, and P = 65520.

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7. $\boxed{165}$ Part A: 99% of the 200 fish = 198 guppies. If you remove 100 guppies, then the remainder is 98 out of 100 total fish or 98%. Part B: Just figure out the times and you will determine that 8:57 is the time. Add the minutes and hour to get 65, then add the parts to get the final answer.

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8. $\boxed{1073}$ Part A: Because the mean and mode are equal, one of the numbers must be repeated, and x must be 40 or 50. If x were 30 or 80, then the median and mode would not be equal. Checking the mean will show that 50 is the correct answer. Part B: Eliminating exponents, the series becomes: $1+2+4+...+512,$ which are the first 9 powers of 2. This sum formula for powers of 2 is $2^{n+1}-1\Rightarrow 2^{10}-1=1023$. Sum the parts to get the final answer.

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9. $\boxed{13}$ Part A: The # of chapters done is directly proportional to the number of people, and directly proportional to the amount of time spent. So, $\frac{\text{(\#of ppl)(amt. of time)}}{\text{\# of chapters}}=k$.
Plugging in that I can do the problems in 1 chapter alone in 8 hours, we get $k=8$. So, if $n$ people can do 30 chapters in 24 hours, and plugging in to the equation, we get $n=10$. Part B: Each good worker can paint 1/12 of my house in an hour, so 3 together can paint 3/12 = ¼ of my house. So, in 3 hours, they paint ¾ of my house. The bad workers have to paint the other ¼ . Each bad worker paints 1/36 of my house in an hour, so in 3 hours each bad worker can paint 1/12 of my house. So, I need (1/4)(1/12) = 3 bad workers. Sum the parts.

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10. $\boxed{8\frac{1}{2}\text{ or }\frac{17}{2}}$ Part A: Clearing fractions gives the equation $2x^2+11x+5=0$ & $x=\frac{-1}{2}$, -5. Their sum is -5.5. Part B: This factors as $(2x^2-1)(x^2-2)$ and $x=\pm\sqrt{2},\ \pm\frac{1}{\sqrt{2}}$. Their sum is 0. Part C: Cross multiply to the get the equation $x^2-12x+32=0$ and $x=4,\ 8$. Their sum is 12. Part D: Factoring gives the equation $\frac{(x-8)(x-1)}{(x-1)}+\frac{(3x-2)(x+1)}{(3x-2)}=-3$ and $-3=2x-7$ and $x=2$. Sum the parts to get the answer.

11. $\boxed{7+27i}$ Part A: We get $-4+27i$. Part B: All sets of 4 consecutive $i$ terms will be 0, and 600 is evenly divisible by 4, hence everything adds to 0 except for the number 1 at the end. So, the sum is 1. Part C: Expanding will give $(5c+18)+(30-3c)i$. Since this product must be real, then $(30-3c)i=0$ and c=10. Sum the parts to get the final answer.

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12. $\boxed{27}$ Part A: Because the train is going 60mph, the front of the train moves 1 m/min. So, in the 3 minutes since the front of the train entered the tunnel, the train has moved 3 miles. At the end of those 3 minutes, the front is 1 mile past the tunnel (the train is 1 mile long and its end is just leaving the tunnel). So, the front has moved 3 miles from the beginning of the tunnel and is now 1 mile beyond the end. Hence, the tunnel is 3 – 1 = 2 miles long. Part B: Since Superman can go all the way around the world in 2.5 hours, he can go $\frac{1}{2.5}$ of the world in 1 hour. The same way, Flash can go around $\frac{1}{1.5}$ of the world in 1 hour. In $x$ hours, Superman travels $\frac{x}{2.5}$ of the world and Flash travels $\frac{x}{1.5}$ of the world. Their paths together make a single path all the way around the world and we get the equation $\frac{x}{2.5}+\frac{x}{1.5}=1$ and $x=\frac{15}{16}$. So, they meet for the 1st time in $\frac{15}{16}$ of an hour and every $\frac{15}{16}$ hours thereafter. Multiplying by 24 hours gives us 25.6 times that they will meet. Disregard the decimal to get 25 times. Sum the parts.

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13. $\boxed{15}$ Part A: Because $f(-3)=2$, we get $81a-9b-3+5=2 \Rightarrow 81a-9b=0$. We also have $f(3)=81a-9b+3+5=0+8=8$. Part B: $f(-4)=41$, and since $f(4)$ also $=41$, and we are given that $g(f(4))=9$, then $g(41)=9$. Part C: Solving $2a\text{-}\frac{6}{a}=-4$ gives $a=1,-3$. Sum the parts.

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14. $\boxed{-1}$ Part L: First, since the vertex is below the x-axis and the parabola crosses the x-axis, then it opens upward and $a$ **must** be positive (1 choice). Part M: You must complete the square to determine the signs of $b$ & $c$. This gives $y=a\left(x^2+\frac{b}{a}x+\frac{b^2}{4a^2}\right)$. Because the vertex is (4, -5), we get $\frac{-b}{2a}=4$ and since $a$ is positive, then $b$ **must** be negative (1 choice). Since the x-coordinates are on opposite sides of the y-axis, one root is positive and one root is negative. So, the product of the 2 roots is negative, which means that $\frac{c}{a}$ must be negative and since $a$ is positive, then $c$ **must** be negative (1 choice). One positive minus 2 negatives = -1.

| | **Theta Bowl Answers** |
|---|---|
| 1. | $\frac{95}{6}$ or $15\frac{5}{6}$ |
| 2. | $-\frac{55}{6}$ or $-9\frac{1}{6}$ |
| 3. | $\frac{9+\sqrt{15}}{2}$ or $4.5+\frac{\sqrt{15}}{2}$ |
| 4. | $\frac{43}{2}$ or $21\frac{1}{2}$ |
| 5. | 1133 |
| 6. | 65,520 |
| 7. | 165 |
| 8. | 1073 |
| 9. | 13 |
| 10. | $\frac{17}{2}$ or $8\frac{1}{2}$ |
| 11. | $7+27i$ |
| 12. | 27 |
| 13. | 15 |
| 14. | $-1$ |