For all questions, answer E. "NOTA" means none of the above answers is correct. Also cis(x) = sin(x) + $i$sin(x) where $i = \sqrt{-1}$.

1. The centrifugal force acting on a mass traveling in a circular path varies directly as the square of its speed and inversely as the radius of the circle. If the centrifugal force is 1,200 pounds when the mass is traveling 40 ft/s along a path 6 ft in diameter, for what speed (in feet per second) will the centrifugal force be 900 lb on the same mass traveling along the same circular path?

    A. $20\sqrt{3}$ B. $20\sqrt{6}$
    C. $10\sqrt{6}$ D. $40\sqrt{3}$ E. NOTA

2. Dudley is going on a road trip from Washington, DC to Dayton, Ohio and back in his Batmobile. Luckily, this is exactly a 500 mile drive. He completed the Washington, DC to St. Louis leg of his trip going 50 mph exactly the entire time. His speedometer died on the way back, but miraculously, Dudley is a genius and knows that his average speed for the entire trip was 60mph. How fast was Dudley going on the way back, assuming he traveled at a constant speed?

    A. 75 mph   B. 300 mph   C. 70 mph   D. 100 mph   E. NOTA

3. The Richter scale measures the intensity level of earthquakes. The Richter scale is a base 10 logarithmic scale. This means that an earthquake that measures 5 on the Richter scale is 10 times more powerful than an earthquake that measures 4 on the scale. On October 1, 1987, there was an earthquake in Whittier, California that registered a 5.9 on the Richter scale. Three days later, a magnitude 5.6 aftershock occurred. How many times stronger was the original earthquake than the aftershock?

    A. 10^3 B. $10^{59/56}$ C. $\sqrt[10]{1000}$ D. 10   E. NOTA

4. The whispering gallery in the Capitol building is in the shape of an ellipse! Because of this, there is an amazing application that occurs. If you and a friend each stand on the location of the one of the two foci of the ellipse, you will be able to hear one another talk while you are far away from each other. Anisha and Paula, who both love to talk, decide to go to the Capitol Building to take part in this phenomenon. Suppose the equation for the whispering gallery is given by:

    $$\frac{x^2}{3600} + \frac{y^2}{10000} = 1$$

    where x, y and all terms of the equation are in feet. How many feet away from each other should Paula and Anisha stand so they can hear each other whisper?

    A. $20\sqrt{34}$ B. $40\sqrt{34}$ C. 80   D. 160   E. NOTA

5. Euler Airlines is trying to determine the best location for their hub airport. Three of their biggest US cities where they fly to are Denver, New York and New Orleans. They want to place their hub equidistant from these three cities. John Q. Thales (the CEO of Euler Airlines) makes a triangle connecting the three cities and determines the location that meets the above criteria. At which point in the triangle is Mr. Thales going to place the hub?

    A. Orthocenter   B. Incenter   C. Circumcenter   D. Centroid   E. NOTA

6. Suppose that vector **R** models the amount of rainfall in inches and the direction in which it falls. Also, vector **A** models the area in square inches and orientation of the opening of a rain gauge. The total volume V of water collected (in cubic inches) in the rain gauge is given by $V = |\mathbf{R} \bullet \mathbf{A}|$. If $\mathbf{R} = \mathbf{i} - 2\mathbf{j}$ and $\mathbf{A} = 0.5\mathbf{i} + \mathbf{j}$, what is the volume of rainwater collected in the gauge in cubic inches?

    A. 1.5   B. 0   C. 2   D. -1.5   E. NOTA

7. To help predict eruptions from the volcano of Mauna Loa on the island of Hawaii, scientists keep track of the volcano's movement using a "super triangle" having vertices at the volcanoes of Mauna Loa, Hualalai and Mauna Kea. For example, Mauna Loa one year moved 6 inches as a result of building up internal pressure. If the distance between Mauna Loa and Mauna Kea is 23 miles, the distance between Mauna Kea and Hualalai is 29 miles and the angle with vertex at Mauna Kea is 43 degrees, what is the distance between Mauna Loa and Hualalai?

    A. $\sqrt{1370 + 1334\cos(43°)}$ B. $\sqrt{1370 - 1334\sin(43°)}$

    C. $\sqrt{1370 - 1334\cos(47°)}$ D. $\sqrt{1370 - 1334\sin(47°)}$ E. NOTA

Use the following information for problems 8-9: There is a rectangular lawn that has a perimeter of 100 meters.

8. Suppose (for this problem), the length of the rectangle is 10 less than 2 times the width. Now suppose you make a garden in the lawn by connecting the midpoints of the sides of the rectangle and planting inside the quadrilateral formed. If a pack of seeds will plant 50 m^2, how many packs will you need to buy to completely fill the garden?

    A. 2   B. 4   C. 8   D. 10   E. NOTA

9. Suppose the lawn was in the shape of a square. Now, you go and mow x% of the lawn, where $0 < x < 100$. The next day, you mow x% of the remaining lawn. The day after, you mow x% of the remaining lawn, and so forth. What percent of the lawn gets mowed after an infinite number of days?

    A. 100%   B. $\left[1 - \left(\dfrac{x}{100}\right)^2\right]\%$

    C. $\left(1 - x^2\right)\%$ D. $\left[\left(\dfrac{x}{100}\right)^2 - 1\right]\%$ E. NOTA

10. For any real numbers x and y, exactly one of the following relations holds:

$$x < y$$
$$x = y$$
$$x > y$$

This statement describes:

A. The Edict of Thales
B. The Rule of Equality
C. The Frisian Corollary
D. The Law of Trichotomy
E. NOTA

11. The current I in a circuit with voltage E, resistance R, capacitive resistance X_C, and inductive resistance X_L is given by:

$$I = \frac{E}{R + (X_L - X_C)i}, \text{ where } i = \sqrt{-1}$$

If $E = 12 cis 30°$, $R = 3$, $X_L = 4$ and $X_C = 6$, determine the value of I.

A. $\dfrac{6(3\sqrt{3} + 2) + 6(2\sqrt{3} - 3)i}{13}$

B. $\dfrac{6(3\sqrt{3} - 2) + 6(2\sqrt{3} + 3)i}{13}$

C. $\dfrac{6(3\sqrt{3} + 2) + 6(2\sqrt{3} - 3)i}{5}$

D. $\dfrac{6(3\sqrt{3} + 2) + 6(2\sqrt{3} - 3)i}{5}$

E. NOTA

Use the following information for problems 12-13: At a viewing party for the "America's Next Top Model" Season Finale, 6 friends are playing a game to determine which two people get to be "Tyra Banks for a day." They are doing this by playing the most boring game in the world – they decide to draw a name from a bag. There are 4 females and 2 males at this particular party.

12. What is the probability a female name gets drawn first and then a male name gets drawn.

A. $\dfrac{2}{9}$ B. $\dfrac{8}{15}$ C. $\dfrac{4}{9}$ D. $\dfrac{4}{15}$ E. NOTA

13. Robert (a male) gets his name chosen first and is just thrilled that he gets the honor of being Tyra for a day! If his name is not placed back in the bag, and the next name is chosen at random, what is the probability the other male at the party will be chosen?

A. 0   B. $\dfrac{1}{5}$ C. $\dfrac{2}{3}$ D. 1   E. NOTA

Use the following information for problems 14-15: A flagpole is centered perpendicular to a rectangular slab of concrete whose length and width are 10 meters and 15 meters. It is tethered to the ground by four wires, each of length 12 meters, and attached from the midpoint of the flagpole to a vertex of the rectangle. The angle the wire makes with the flagpole will be known as θ.

14. How tall is the flagpole in meters?

A. $\dfrac{5\sqrt{13}}{4}$ B. $\dfrac{5\sqrt{13}}{2}$ C. $\dfrac{\sqrt{251}}{4}$ D. $\dfrac{\sqrt{251}}{2}$ E. NOTA

15. Which of the following trigonometric statements is correct?

A. $\csc\theta = \dfrac{24\sqrt{251}}{251}$ B. $\sin\theta = \dfrac{24\sqrt{13}}{65}$

C. $\cot\theta = \dfrac{\sqrt{3263}}{65}$ D. $\tan\theta = \dfrac{\sqrt{251}}{251}$ E. NOTA

16. Joe is an artist of geometric art. His newest creation is going to be made by building a tessellation on his canvas. While Joe is not sure which shapes he will be using to make his art, which of the following is NOT going to be an option for him to use?

A. Isosceles Trapezoids   B. Regular Octagons and Squares
C. Regular Hexagons   D. Regular Pentagons   E. NOTA

17. In physics, destructive interference refers to the canceling of two waves. This occurs whenever the two waves intersect each other on a certain interval. Suppose one wave has equation $y = 2\sin(2x)$ and a second wave has equation $y = 3\cos(x)$. How many times will destructive interference occur with these two waves on the interval $[0,2\pi]$?

A. 2   B. 4   C. 6   D. 8   E. NOTA

18. If two resistors, R_1 and R_2, are connected in *parallel* in an electric circuit, the net resistance, R, is given by $\dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$. If R_1=9 ohms, what values of R_2, in terms of ohms, will result in a net resistance of less than 8 ohms? Assume that resistance is only positive.

A. $(-\infty,-72]$ B. $(-\infty,72]$ C. $[-72,\infty)$ D. $[72,\infty)$ E. NOTA

19. The hour hand on Big Ben in London is 9 feet long. What is the length of the arc swept by the tip of the hour hand from 3:00 PM to 3:40 PM (in feet)?

A. $\dfrac{23\pi}{18}$ B. π   C. $\dfrac{7\pi}{6}$ D. 12π E. NOTA

Use the following information for problems 20-21: Math is critically important in the planning of many networks, including road construction. If roads are planned as *Euler Circuits* it is possible for someone to traverse every edge <u>exactly once</u> and start and end at the same spot! This greatly reduces the time needed to return to the same spot while still travel along all of the roads in the network.

20. Marisol the meter maid is very knowledgeable in mathematics and chooses her routes carefully based on whether or not they represent Euler Circuits. By doing this, she is able to park at one location, walk all of the streets and end at the same location she began – not wasting any unnecessary time or energy. If Marisol is given the option of the following road systems, which should she pick if she only picks networks that represent Euler Circuits?

A.   B.   C.   D.   E. NOTA
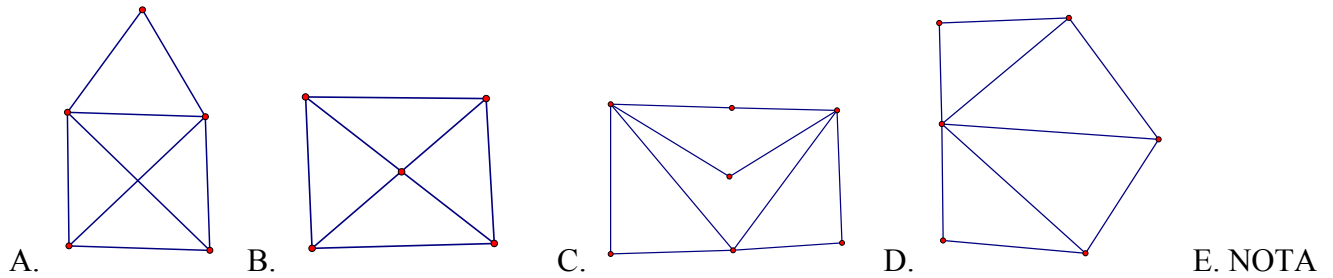
21. A few months later, Marisol is given a new assignment. However, she was not given the option of choosing her route. She is given the route below. Marisol notices that the network does not represent an Euler Circuit and knows that by traveling this route she will be wasting energy. She plans on going to her boss to suggest changing the number of roads in her route to less than the present route so that it can become an Euler Circuit. What is the least number of roads that can be removed so that Marisol can make the path an Euler Circuit?

A. 0   B. 1   C. 2   D. 3   E. NOTA
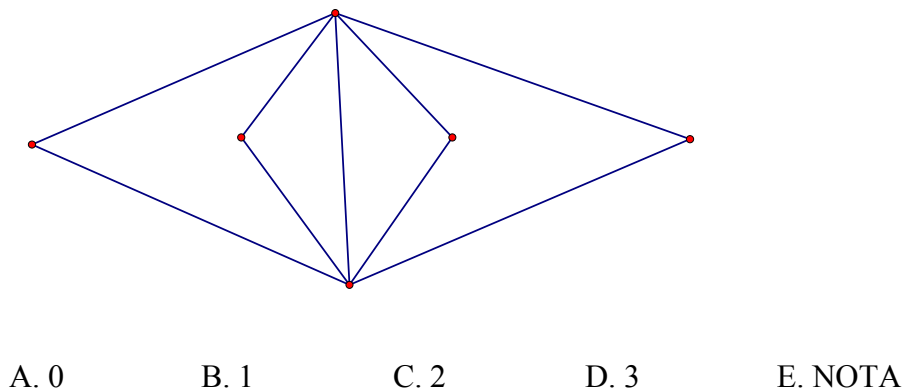
22. Newton's Law of Cooling models the cooling (or heating) of an object placed in a surrounding medium (for example, a hot pizza that is taken out of the oven and allowed to cool on the kitchen counter). Newton's Law is defined as $u(x) = T + (I - T)e^{kx}$, where

$u(x)$ = Temperature at time x (where x is defined as the minutes elapsed since time x = 0)
I = Initial Temperature
T = Temperature of the surrounding medium
K = a cooling/heating constant determined by the nature of the object

A can of Mountain Dew is left out on a table for 10 minutes. If the Mountain Dew was 35°F to start, the temperature of the room was 70°F, and the temperature after the ten minutes had elapsed is 40°F, then according to Newton's Law of Cooling, what is Mountain Dew's constant of heating?

A. $10\ln(\frac{6}{7})$　　　　B. $\frac{1}{10}\ln(\frac{6}{7})$　　　　C. $\frac{1}{10}\ln(\frac{7}{6})$　　　　D. $10\ln(\frac{7}{6})$　　E. NOTA

23. An ecologist goes to Lake Okeechobee to run an experiment to determine the number of fish present in the lake. She initially goes out on day 1 and randomly tags 120 fish. She goes out on day 2 and collects 550 fish, of which 80 have been tagged the previous day. Using this information, what is the best estimate for the total number of fish in the lake?

A. 367　　　　B. 590　　　　C. 825　　　　D. 200　　　　E. NOTA

Use the following information for problems 24-25: An air traffic controller in a radar station is using a polar coordinate system to track the route of Waldo Air Flight 1340. At 1:30pm, the plane was exactly 4 miles away from the radar station and was 60 degrees east of north. At 1:40pm, the plane was exactly 6 miles away from the radar station and was 60 degrees west of north.

24. Assuming the plane travels in a linear route, what is the rectangular equation of the line describing the plane's path? (Assume x, y and all constants are in miles)

A. $y = \frac{x\sqrt{3}}{3} + 6$　　　　B. $y = \frac{-x\sqrt{3}}{18} + 3$

C. $y = \frac{-x\sqrt{3}}{15} + \frac{12}{5}$　　　　D. $y = \frac{-x\sqrt{3}}{15} + \frac{18}{5}$　　　E. NOTA

25. Assuming the plane travels in a linear route, what is the polar equation of the line describing the plane's path?

A. $r = \dfrac{36}{15\sin\theta - \sqrt{3}\cos\theta}$　　　　B. $r = \dfrac{54}{15\sin\theta - \sqrt{3}\cos\theta}$

C. $r = \dfrac{54}{15\sin\theta + \sqrt{3}\cos\theta}$　　　　D. $r = \dfrac{36}{15\sin\theta + \sqrt{3}\cos\theta}$　　　E. NOTA

26. On the picture below (not to scale, adapted from an actual Chinese drawing from around 1000 B.C.) the area of the large square ABCD is 25, and the area of the small square A'B'C'D' is 1. What is the length AA'?
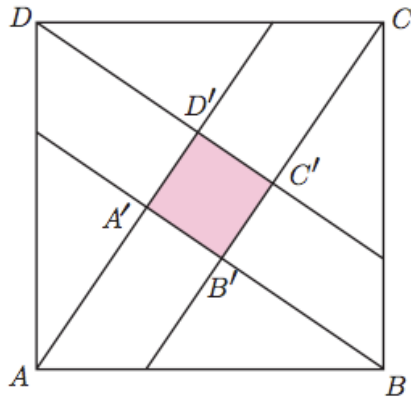
A. 3   B. $2\sqrt{2}$ C. $2\sqrt{3}$ D. 2   E. NOTA

27. Global Recycling Company recycles aluminum widgets by melting them down and molding them into aluminum doodads. Aluminum widgets are in the shape of a cone, with the base diameter of 4 centimeters and a slant height of $2\sqrt{5}$ centimeters. Aluminum doodads are in the shape of spheres with a radius of 1 centimeter. If the recycling process turns 100 aluminum widgets into exactly 150 aluminum doodads, what percent of aluminum is lost in the recycling process?

A. 81.25%   B. 37.5%   C. 62.5%   D. 18.75%   E. NOTA

Use the following information for problems 28-30: The IS/LM model is a macroeconomic tool that demonstrates the relationship between interest rates and real output in the goods and services market and the money market. The model consists of two curves: the IS curve and the LM curve. The IS curve is determined by the formula $Y = C + I + G$ where Y represents income, C represents consumption, I represents investment and G represents government purchases. The LM curve is determined by the formula MS = MD where MS represents the money supply and MD represents the money demand. Income (Y) boils down to a function of r, where r represents interest, measured in percent. For example, if $r = 6$, the interest rate would be 6%. Assume the following model of the economy:

$$C = 0.8(Y - T) \qquad\qquad I = 800 - 20r$$
$$MS = 1200 \qquad\qquad T = 1000$$
$$G = 1000 \qquad\qquad MD = 0.4Y - 40r$$

28. Which of the following is a numerical formula for the IS curve, showing Y as a function of r alone?

A. $Y = 3000 + 100r$ \qquad B. $Y = 5000 - 100r$

C. $Y = 480 + 16r$ \qquad D. $Y = 200 - 4r$ \qquad E. NOTA

29. Which of the following is a numerical formula for the LM curve, showing Y as a function of r alone?

A. $Y = 3000 + 100r$ \qquad B. $Y = 5000 - 100r$

C. $Y = 480 + 16r$ \qquad D. $Y = 200 - 4r$ \qquad E. NOTA

30. If government purchases were to double, in which direction would the IS curve shift?

A. Up \qquad B. Left \qquad C. Down \qquad D. Right \qquad E. NOTA