1. B

$i\sqrt{i\sqrt{\dots}} = c$

$i\sqrt{c} = c$

$-c = c^2$

$c = 0, -1$

$i\sqrt{i\sqrt{\dots}} \neq 0$

$i\sqrt{i\sqrt{\dots}} = -1$

2. C

$d = \sqrt{(4-11)^2 + (-7-17)^2}$

$d = \sqrt{49+576} = \sqrt{625} = 25$

3. A

$\sqrt{-4} \times \sqrt{-3} \quad \underline{?} \quad \sqrt{(-4)(-3)}$

$(2i) \times i\sqrt{3} \quad \underline{?} \quad \sqrt{12}$

$-2\sqrt{3} \quad \underline{<} \quad 2\sqrt{3}$

4. B

$|4i| \times |3i| \quad \underline{?} \quad |4i \times 3i|$

$4 \times 3 \quad \underline{?} \quad |-12|$

$12 \quad \underline{=} \quad 12$

5. B

Let $z_1 = a + bi$ and let $z_2 = c + di$.

$$p(x) = (x - (a+bi))(x - (c+di))$$
$$= x^2 - x((a+bi) + (c+di)) + (a+bi)(c+di)$$
$$= x^2 - x(a + c + (b+d)i) + ac - bd + (bc + ad)i$$

$b + d = 0$

$b = -d$

$bc + ad = 0$

$bc - ab = 0$

$b(c-a) = 0$

$b \neq 0, \ \therefore \ c - a = 0$

$c = a$

Therefore, when $z_1 = a + bi,\ z_2 = c + di = a - bi$.

Also, if $z_1 = a + bi$ and $z_2 = \overline{z_1} = a - bi$

$$p(x) = (x - (a+bi))(x - (a-bi))$$
$$= x^2 - x((a+bi) + (a-bi)) + (a+bi)(a-bi)$$
$$= x^2 - 2ax + (a^2 + b^2)$$

Therefore, if $z_1 = \overline{z_2}$ then the coefficients of $p(x)$ are real, and if the coefficients of $p(x)$ are real then $z_1 = \overline{z_2}$.

Counterexample: $1+i, \quad -1-i \rightarrow x^2 - 2i$
(disproves all incorrect answers)

6. B

$$\frac{\left[4\operatorname{cis}\left(\frac{\pi}{2}\right)\right]^3 \left[\sqrt{2}\operatorname{cis}\left(\frac{11\pi}{6}\right)\right]}{\sqrt{8\operatorname{cis}\left(\frac{2\pi}{3}\right)}} = \frac{\left[64\operatorname{cis}\left(\frac{3\pi}{2}\right)\right]\left[\sqrt{2}\operatorname{cis}\left(\frac{11\pi}{6}\right)\right]}{2\sqrt{2}\operatorname{cis}\left(\frac{\pi}{3}\right)} =$$

$$= \frac{64 \times \sqrt{2}}{2\sqrt{2}}\operatorname{cis}\left(\frac{3\pi}{2} + \frac{11\pi}{6} - \frac{\pi}{3}\right) = 32\operatorname{cis}(3\pi) = -32$$

7. C

$$re^{i\theta} = \sqrt[4]{-1}$$
$$r^4e^{4i\theta} = -1$$
$$r^4e^{4i\theta} = 1\text{cis}(\pi + 2\pi k)$$
$$r^4e^{4i\theta} = 1e^{i\pi + 2\pi k}$$
$$r = 1$$
$$4\theta = \pi + 2\pi k$$
$$\theta = \frac{\pi}{4} + \frac{\pi}{2}k$$
$$re^{i\theta} = e^{i\left(\frac{\pi}{4}+\frac{\pi}{2}k\right)} = \pm\frac{\sqrt{2}}{2} \pm \frac{\sqrt{2}}{2}i$$
$$|ab| = \left|\left(\pm\frac{\sqrt{2}}{2}\right)\left(\pm\frac{\sqrt{2}}{2}\right)\right| = \frac{1}{2}$$

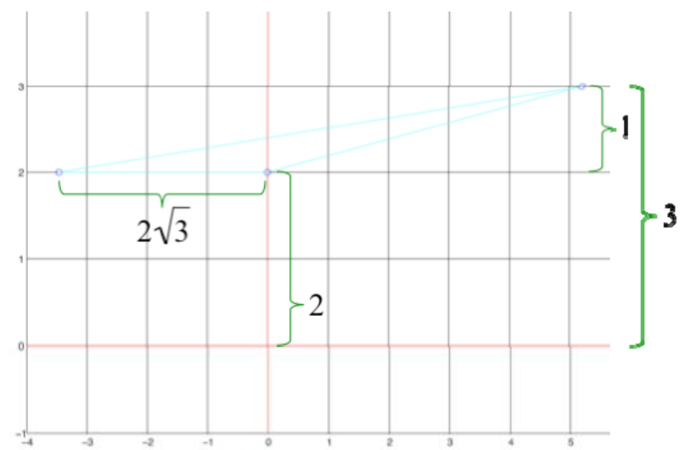
8. C

$$re^{i\theta} = \sqrt[5]{-6-2i\sqrt{3}}$$
$$\left(re^{i\theta}\right)^5 = \left(\sqrt[5]{-6-2i\sqrt{3}}\right)^5$$
$$r^5e^{i5\theta} = 4\sqrt{3}\left(-\frac{\sqrt{3}}{2} - \frac{1}{2}i\right)$$
$$r^5e^{i5\theta} = 4\sqrt{3}\left(-\frac{\sqrt{3}}{2} - \frac{1}{2}i\right)$$
$$r^5e^{i5\theta} = 4\sqrt{3}\text{cis}\left(\frac{7\pi}{6} + 2\pi k\right)$$
$$5\theta = \frac{7\pi}{6} + 2\pi k$$
$$\theta = \frac{7\pi + 12\pi k}{30}$$
$$\theta = \frac{7\pi}{30}, \frac{19\pi}{30}, \frac{31\pi}{30}, \frac{43\pi}{30}, \frac{55\pi}{30}$$
$$\frac{7\pi}{30} + \frac{19\pi}{30} + \frac{31\pi}{30} + \frac{43\pi}{30} + \frac{55\pi}{30} = \frac{31\pi}{6}$$

9. A

$$z = a + 3i$$
$$z^2 = a^2 - 9 + 6ai$$
$$z^3 = a^3 - 27a + i(9a^2 - 27)$$
$$\text{Re}(z^2) = a^2 - 9$$
$$\text{Im}(z^3) = 9a^2 - 27$$
$$a^2 - 9 = 9a^2 - 27$$
$$a^2 = \frac{18}{8}$$
$$|\text{Re}(z)| = |a| = \left|\pm\frac{3}{2}\right| = \frac{3}{2}$$

10. B



$$A = \frac{1}{2}bh = \frac{1}{2}\left(2\sqrt{3}\right)(1) = \sqrt{3}$$

11. C

I. $3\sqrt{3}\text{cis}(60°)$

II. $3\sqrt{3}e^{i\frac{7\pi}{3}} = 3\sqrt{3}e^{i\frac{7\pi}{3}-2\pi} = 3\sqrt{3}e^{i\frac{\pi}{3}} = 3\sqrt{3}\text{cis}(60°)$

III. $\frac{9}{2} + \frac{3\sqrt{3}}{2}i = 3\sqrt{3}\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right) = 3\sqrt{3}\text{cis}(30°)$

IV. $3\sqrt{3}\left(\sin\left(\frac{\pi}{6}\right) + i\cos\left(\frac{\pi}{6}\right)\right) = 3\sqrt{3}\left(\cos\left(\frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{3}\right)\right) =$

$= 3\sqrt{3}\text{cis}(60°)$

$\text{I} = \text{II} = \text{IV} \neq \text{III}$

12. C

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x_1-x_2=\frac{-b+\sqrt{b^2-4ac}}{2a}-\frac{-b-\sqrt{b^2-4ac}}{2a}=\frac{\sqrt{b^2-4ac}}{a}$$

$$\frac{\sqrt{b^2-4ac}}{a}=\frac{\sqrt{(2i)^2-4(1)\left(-i\sqrt{3}\right)}}{1}=\sqrt{-4+4i\sqrt{3}}$$

$$\left|\sqrt{-4+4i\sqrt{3}}\right|=\left|\sqrt{8\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)}\right|=\left|\left(8\operatorname{cis}\frac{2\pi}{3}\right)^{1/2}\right|=2\sqrt{2}$$

13. A

$$C_{32}=(-1)^{3+2}\begin{vmatrix}4i & -1 & 0\\ 6 & 3 & 1\\ -5i & i & -2i\end{vmatrix}$$

$$=-\left[(24+5i+0)-(0-4+12i)\right]$$

$$=-\left[28-7i\right]=-28+7i$$

$$\det(A)=a_{ij}C_{ij}$$

$$\det(A)=a_{32}C_{32}$$

$$\det(A)=(1)(-28+7i)=-28+7i$$

14. B

If $f(x)$ has only real coefficients, then for any root with a non-zero imaginary component, the conjugate of that root must also be a solution to $f(x)=0$.

$$f(x)=\left(x-(-3i)\right)\left(x-(3i)\right)\left(x-(5+2i)\right)\left(x-(5-2i)\right)$$

$$=\left(x^2+9\right)\left(x^2-10x+29\right)$$

$$=x^4-10x^3+38x^2-90x+261$$

$$a+b+c+d+e=1-10+38-90+261=200$$

15. B

$$(2i-9)z=n+0i$$

$$(2i-9)(a+bi)=n+0i$$

$$-9a-2b+i(2a-9b)=n+0i$$

$$2a-9b=0$$

$$a=\frac{9b}{2}$$

$$-9a-2b=n$$

$$-9\left(\frac{9b}{2}\right)-2b=n$$

$$-85b=2n$$

16. C

$$4z-3i\bar{z}=10+3i$$

$$4(a+bi)-3i(a-bi)=10+3i$$

$$4a-3b+i(-3a+4b)=10+3i$$

$$4a-3b=10 \quad -3a+4b=3$$

$$a=\frac{\begin{vmatrix}10 & -3\\ 3 & 4\end{vmatrix}}{\begin{vmatrix}4 & -3\\ -3 & 4\end{vmatrix}}=\frac{40+9}{16-9}=7 \quad b=\frac{\begin{vmatrix}4 & 10\\ -3 & 3\end{vmatrix}}{\begin{vmatrix}4 & -3\\ -3 & 4\end{vmatrix}}=\frac{12+30}{16-9}=6$$

$$a-b=7-6=1$$

17. D

Let $z=x+yi$

$$\operatorname{Re}\left((x+yi)+(x-yi)^2\right)=2$$

$$\operatorname{Re}\left(x+yi+x^2-y^2-2xyi\right)=2$$

$$x^2+x-y^2=2$$

$$\left(x+\frac{1}{2}\right)^2-y^2=\frac{9}{4}$$

$$\frac{\left(x+\frac{1}{2}\right)^2}{4/9}-\frac{y^2}{4/9}=1$$

18. A

$$1-i=\sqrt{2}\operatorname{cis}\left(-\frac{\pi}{4}\right)$$

$$\sqrt{6}\operatorname{cis}\left(\frac{\pi}{4}\right)=\sqrt{3}+i\sqrt{3}$$

$$\frac{2\left[\sqrt{2}\operatorname{cis}\left(-\frac{\pi}{4}\right)\right]\left[\sqrt{6}\operatorname{cis}\left(\frac{\pi}{4}\right)\right]}{(1-i)+\left(\sqrt{3}+i\sqrt{3}\right)}=\frac{4\sqrt{3}\operatorname{cis}(0)}{\left(1+\sqrt{3}\right)+i\left(-1+\sqrt{3}\right)}$$

$$\arctan\left(\frac{-1+\sqrt{3}}{1+\sqrt{3}}\right)=\arctan\left(2-\sqrt{3}\right)=15°$$

$$c\cos(15°)=1+\sqrt{3}$$

$$c=\left(1+\sqrt{3}\right)\left(\frac{4}{\sqrt{6}+\sqrt{2}}\right)=\left(1+\sqrt{3}\right)\left(\frac{4}{\sqrt{6}+\sqrt{2}}\right)\left(\frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)$$

$$c=\left(1+\sqrt{3}\right)\left(\sqrt{6}-\sqrt{2}\right)=\sqrt{2}$$

$$\frac{4\sqrt{3}\operatorname{cis}(0)}{\sqrt{2}\operatorname{cis}(15°)}=2\sqrt{6}e^{-i15°}$$

19. A

$$\binom{7}{6-1}(2x)^2(-i)^{6-1}=\frac{7!}{5!2!}\left(4x^2\right)(-1)^5(i)^5$$

$$=\frac{7\times 6}{2\times 1}\left(4x^2\right)(-1)(i)$$

$$=-84ix^2$$

20. C

For $y=2$,

$$\left(8x^3+27\right)(2)^2-8x^3+4=31$$

$$32x^3+108-8x^3+4=31$$

$$x^3=-\frac{81}{24}=-\frac{27}{8}$$

$$\left(re^{i\theta}\right)^3=-\frac{27}{8}$$

$$r^3=\frac{81}{16}=\frac{3}{2}$$

$$e^{3i\theta}=e^{i(\pi+2\pi k)}$$

$$3\theta=\pi+2\pi k$$

$$\theta=\frac{\pi}{3}+\frac{2\pi}{3}k$$

$$x=\frac{3}{2}\operatorname{cis}\frac{\pi}{3},\quad \frac{3}{2}\operatorname{cis}\pi,\quad \frac{3}{2}\operatorname{cis}\frac{5\pi}{3}$$

$$x=\frac{3}{4}+\frac{3\sqrt{3}}{4}i,\quad -\frac{3}{2},\quad \frac{3}{4}-\frac{3\sqrt{3}}{4}i$$

21. A

$$(a+bi)^2=9-40i$$

$$a^2-b^2+2abi=9-40i$$

$$a^2-b^2=9 \qquad 2ab=-40$$

$$a^2=9+b^2 \qquad b\sqrt{9+b^2}=-20$$

$$b^2\left(9+b^2\right)=400$$

$$b^4+9b^2-400=0$$

$$(b^2-16)(b^2+25)=0$$

$$b=\{\pm 4,\pm 5i\},\ b<0 \quad\rightarrow\quad b=-4$$

$$a^2=9+(-4)^2=25$$

$$a=\pm 5,\ a>0 \quad\rightarrow\quad a=5$$

$$a+b=5+(-4)=1$$

22. A

Because $g(x)$ is a 6th degree polynomial, there are 6 complex roots of $g(x)$. I is therefore true.

By Descartes' rule of signs, there are either 3 or 1 positive real roots and either 3 or 1 negative real roots. The chart below shows all possible combinations of the nature of the roots:

Positive Real Roots	Negative Real Roots	Total Real Roots	Total Complex with Imaginary Component
3	3	6	0
3	1	4	2
1	3	4	2
1	1	2	4

Therefore, II cannot be true, III cannot be true, and IV cannot be true.

23. D was changed to E at the convention by the resolution center.

If $z_1 = 2e^{i\pi}$ and $z_2 = 4e^{i\pi}$,

$\left|2e^{i\pi} + 4e^{i\pi}\right| \underline{\ ?\ } \left|2e^{i\pi}\right| + \left|4e^{i\pi}\right|$

$\left|6e^{i\pi}\right| \underline{\ ?\ } 2 + 4$

$6 = 6$

$|z_1 + z_2|$ can be equal to $|z_1| + |z_2|$.

Therefore, I is false.

$\left|\frac{r_1 e^{i\theta}}{r_2 e^{i\phi}}\right| = \left|r_1 e^{i\theta}\right| \div \left|r_2 e^{i\phi}\right|$

$\left|\frac{r_1}{r_2} e^{i(\theta-\phi)}\right| = r_1 \div r_2$

$\frac{r_1}{r_2} = \frac{r_1}{r_2}$

Therefore, II is true.

$\overline{z_1 - z_2} \underline{\ ?\ } \overline{z_1} + \overline{z_2}$

$z_1 = a + bi \quad z_2 = c + di$

$\overline{(a+bi)-(c+di)} \underline{\ ?\ } \overline{(a+bi)} + \overline{(c+di)}$

$\overline{(a-c)+(b-d)i} \underline{\ ?\ } (a-bi)+(c-di)$

$(a-c)-(b-d)i \underline{\ ?\ } (a+c)+(-b-d)i$

$(a-c)+(-b+d)i \neq (a+c)+(-b-d)i$

Therefore, III is false.

$z\overline{z} \underline{\ ?\ } |z|^2$

$z = a + bi$

$(a+bi)(a-bi) \underline{\ ?\ } |(a+bi)|^2$

$a^2 + b^2 \underline{\ ?\ } \left(\sqrt{a^2+b^2}\right)^2$

$a^2 + b^2 = a^2 + b^2$

Therefore, IV is true.

24. C

By DeMoivre's Theorem, for $z = r(\cos\theta + i\sin\theta)$, $z^n = r^n(\cos n\theta + i\sin n\theta)$. Therefore,

$$\left[\sqrt{2}\operatorname{cis}\left(\frac{\pi}{9}\right)\right]^6 = \left(\sqrt{2}\right)^6 \operatorname{cis}\left(6\times\frac{\pi}{9}\right) = 8\operatorname{cis}\left(\frac{2\pi}{3}\right).$$

25. B

$$f(x) = \frac{1}{x^2 - x}$$

$$f\left(\frac{1-i\sqrt{3}}{2}\right) = \frac{1}{\left(\frac{1-i\sqrt{3}}{2}\right)^2 - \left(\frac{1-i\sqrt{3}}{2}\right)}$$

$$f\left(\frac{1-i\sqrt{3}}{2}\right) = \frac{1}{\left(\frac{-2-2i\sqrt{3}}{4}\right) - \left(\frac{1-i\sqrt{3}}{2}\right)}$$

$$f\left(\frac{1-i\sqrt{3}}{2}\right) = \frac{2}{-2} = -1$$

26. A

If point 5 is located at $re^{i\theta}$ where $r > 1$ and $2\pi k < \theta < \frac{\pi}{2} + 2\pi k$, the reciprocal of point 5 is equal to $\frac{1}{r}e^{-i\theta}$. Since $r > 1$, $\frac{1}{r} < 1$; and since $2\pi k < \theta < \frac{\pi}{2} + 2\pi k$, $-2\pi k > -\theta > -\frac{\pi}{2} - 2\pi k$. Therefore, the reciprocal of point 5 must be inside the unit circle in the quadrant IV, and the only point that satisfies this criteria is point 4.

27. A

$$\begin{vmatrix} 4-\lambda & 2 & 4 \\ 2 & 1-\lambda & 1 \\ -4 & -1 & -3-\lambda \end{vmatrix}$$

$$= \left[(4-\lambda)(1-\lambda)(-3-\lambda) - 8 - 8\right] - \left[-16(1-\lambda) - (4-\lambda) + 4(-3-\lambda)\right]$$

$$= \left[\left(-12 + 11\lambda + 2\lambda^2 - \lambda^3\right) - 16\right] + \left[16 - 16\lambda + 4 - \lambda + 12 + 4\lambda\right]$$

$$= -\lambda^3 + 2\lambda^2 - 2\lambda + 4$$
$$= -\lambda^2(\lambda - 2) - 2(\lambda - 2)$$
$$= -\left(\lambda^2 + 2\right)(\lambda - 2)$$

$$\lambda = \pm 2i,\ 2$$
$$\lambda_1 = a_1 + b_1 i = 0 + 2i$$
$$\lambda_2 = a_2 - b_2 i = 0 - 2i$$
$$\left|\frac{a_1 + a_2}{b_1 + b_2}\right| = \left|\frac{0+0}{2+2}\right| = \left|\frac{0}{4}\right| = 0$$

28. A

$$Z = 2i = 2e^{i90^\circ}$$
$$V = 8e^{i50^\circ}$$
$$V = IZ$$
$$I = \frac{V}{Z} = \frac{8e^{i50^\circ}}{2e^{i90^\circ}} = 4e^{-i40^\circ}$$

29. A

$$P_{\text{avg}} = \frac{1}{2}|V||I|\cos\phi$$
$$= \frac{1}{2}\left|8e^{i50^\circ}\right|\left|4e^{-i40^\circ}\right|\cos(50 - (-40))$$
$$= \frac{1}{2}(8)(4)\cos(90)$$
$$= \frac{1}{2}(8)(4)(0)$$
$$= 0$$

30. B

$$Z = 3\sqrt{3} - 3i = 6e^{-i30°}$$

$$I = 3e^{i25°}$$

$$V = IZ$$

$$V = \left(3e^{i25°}\right)\left(6e^{-i30°}\right) = 18e^{-i5°}$$

$$P_{\text{avg}} = \frac{1}{2}|V||I|\cos\phi$$

$$= \frac{1}{2}\left|18e^{-i5°}\right|\left|3e^{i25°}\right|\cos((-5)-25)$$

$$= \frac{1}{2}(18)(3)\cos(-30)$$

$$= \frac{1}{2}(18)(3)\left(\frac{\sqrt{3}}{2}\right)$$

$$= \frac{27\sqrt{3}}{2}$$