For all questions, answer E. "NOTA" means none of the above answers is correct.
For complex number $z = a + bi$; $\text{Re}(z) = a$, $\text{Im}(z) = b$, and $\bar{z}$ denotes the conjugate of z.
$\text{cis}(\theta) = \cos(\theta) + i\sin(\theta)$.

1. For the non-zero expression $i\sqrt{i\sqrt{\ldots}}$, which is equivalent?

A. i B. –1 C. $-i$ D. 1 E. NOTA

2. Find the distance between $4 - 7i$ and $11 + 17i$ in the complex plane.

A. 5 B. 17 C. 25 D. 31 E. NOTA

3. $\sqrt{-4} \times \sqrt{-3}$? $\sqrt{(-4)(-3)}$

A. < B. = C. > E. NOTA

4. $|4i| \times |3i|$? $|4i \times 3i|$

A. < B. = C. > E. NOTA

5. Given a quadratic equation $p(x)$ whose roots are non-real numbers z_1 and z_2, the coefficients of $p(x)$ are real if and only if

A. $|z_1| = |z_2|$ B. $z_1 = \overline{z_2}$ C. $z_1 = -z_2$ D. $\text{Im}(z_1) = -\text{Im}(z_2)$ E. NOTA

6. Simplify:

$$\frac{\left[4\text{cis}\left(\frac{\pi}{2}\right)\right]^3 \left[\sqrt{2}\,\text{cis}\left(\frac{11\pi}{6}\right)\right]}{\sqrt{8\text{cis}\left(\frac{2\pi}{3}\right)}}$$

A. $-32i$ B. –32 C. –6 D. $6i$ E. NOTA

7. If $\sqrt[4]{-1} = \pm a \pm bi$ where a and b are positive real numbers, what is ab?

A. 0 B. $\frac{\sqrt{3}}{4}$ C. $\frac{1}{2}$ D. 1 E. NOTA

8. Given that any complex number equivalent to $\sqrt[5]{-6 - 2i\sqrt{3}}$ can be written in the form $re^{i\theta}$ where $r > 0$, $0 \le \theta < 2\pi$, and r, $\theta \in \mathbb{R}$, find the sum of all possible values of θ.

A. $\frac{25\pi}{6}$ B. $\frac{67\pi}{15}$ C. $\frac{31\pi}{6}$ D. $\frac{16\pi}{3}$ E. NOTA

9. Let z represent a complex number. If $\text{Re}(z^2) = \text{Im}(z^3)$ and $\text{Im}(z) = 3$, find $|\text{Re}(z)|$.

A. 1.5 B. $\frac{3\sqrt{10}}{5}$ C. 2.25 D. 3 E. NOTA

10. The complex numbers $6e^{i30^\circ}$, $2e^{i90^\circ}$, and $4e^{i150^\circ}$ form a triangle in the complex plane. Find the area of this triangle.

A. $\frac{\sqrt{3}}{2}$ B. $\sqrt{3}$ C. $2\sqrt{3}$ D. $4\sqrt{3}$ E. NOTA

11. Which of the following expressions are equivalent?

I. $3\sqrt{3}\text{cis}(60^\circ)$ II. $3\sqrt{3}e^{i\frac{7\pi}{3}}$

III. $\frac{9}{2}+\frac{3\sqrt{3}}{2}i$ IV. $3\sqrt{3}\left(\sin\left(\frac{\pi}{6}\right)+i\cos\left(\frac{\pi}{6}\right)\right)$

A. I, II, III only B. I, II only C. I, II, IV only D. I, III, IV only E. NOTA

12. For the equation $x^2+2ix-i\sqrt{3}=0$, which of the following is equivalent to the absolute value of the difference between the two roots?

A. $\sqrt{2}$ B. 2 C. $2\sqrt{2}$ D. $4\sqrt{2}$ E. NOTA

13. Evaluate:

$$\begin{vmatrix} 4i & 43 & -1 & 0 \\ 6 & -2i & 3 & 1 \\ 0 & 1 & 0 & 0 \\ -5i & 17i & i & -2i \end{vmatrix}$$

A. $-28+7i$ B. $-20+17i$ C. $20-17i$ D. $28-7i$ E. NOTA

14. The polynomial $f(x)=x^4+ax^3+bx^2+cx+d$ has real coefficients. If $f(-3i)=f(5+2i)=0$, what is $a+b+c+d$?

A. 198 B. 199 C. 200 D. 201 E. NOTA

15. If *S* is the set of points *z* in the complex plane such that $(2i-9)z$ is a real number, then *S* is

A. a point. B. a line. C. a parabola. D. a triangle. E. NOTA

16. The complex number *z* that satisfies $4z-3i\bar{z}=10+3i$ can be written in the form $a+bi$ where *a* and *b* are real numbers. Find $a-b$.

A. –1 B. 0 C. 1 D. 2 E. NOTA

17. What conic section is represented by the equation $\text{Re}\left(z+\bar{z}^2\right)=2$?

A. circle B. ellipse C. parabola D. hyperbola E. NOTA

18. Find the harmonic mean of $1-i$ and $\sqrt{6}\text{cis}\left(\frac{\pi}{4}\right)$. *Hint:* $\tan(15°) = 2-\sqrt{3}$.

A. $2\sqrt{6}e^{-i15°}$ B. $\sqrt{3}e^{-i15°}$ C. $\sqrt{6}e^{i15°}$ D. $2\sqrt{3}e^{i15°}$ E. NOTA

19. Find the coefficient of the 6th term in the binomial expansion of $(2x-i)^7$.

A. –84*i* B. –84 C. 84 D. 84*i* E. NOTA

20. Which of the following ordered pairs (x,y) is a solution to the following equation when $y = 2$?

$$(8x^3+27)y^2 - 8x^3 + 4 = 31$$

A. $(6+7i\,,\,2)$ B. $(6+7i\sqrt{2}\,,\,2)$ C. $\left(\frac{3}{4}-\frac{3\sqrt{3}}{4}i\,,\,2\right)$ D. $\left(-\frac{3\sqrt{2}}{4}i\,,\,2\right)$ E. NOTA

21. If $x^2 = 9-40i$, then a complex solution for *x* is $a+bi$ where $b<0<a$. Find $a+b$.

A. 1 B. 2 C. 3 D. 4 E. NOTA

22. How many of the following statements are true concerning the polynomial $g(x) = x^6 + ax^5 - bx^4 - cx^3 + dx^2 + ex - f$ if *a*, *b*, *c*, *d*, *e*, and *f* are positive real numbers?

I. There are 6 complex roots of $g(x)$.

II. There could be exactly 3 real roots of $g(x)$.

III. All real roots of $g(x)$ could be positive.

IV. All 6 roots of $g(x)$ could be non-real.

A. 1 B. 2 C. 3 D. 4 E. NOTA

23. Which of the following are always true for complex numbers *z*, z_1, and z_2?

I. $|z_1+z_2| < |z_1|+|z_2|$ II. $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$

III. $\overline{z_1-z_2} = \overline{z_1}+\overline{z_2}$ IV. $z\bar{z} = |z|^2$

A. II only B. II, III only C. I, III only D. II, IV only E. NOTA

24. By whose theorem must the following statement be true?

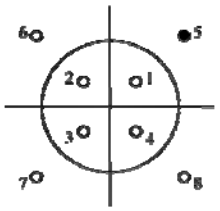
$$\left[\sqrt{2}\text{cis}\left(\frac{\pi}{9}\right)\right]^6 = 8\text{cis}\left(\frac{2\pi}{3}\right)$$

A. Euclid B. Euler C. DeMoivre D. DeMorgan E. NOTA

25. If $f(x)=\dfrac{1}{x^2-x}$, find $f\left(\dfrac{1-i\sqrt{3}}{2}\right)$.

A. –2 B. –1 C. 1 D. 2 E. NOTA

26. The drawing shows several numbered points in the complex plane. The circle is a unit circle centered at the origin. Add the numbers associated with the points that could possibly be the reciprocal of point 5.



A. 4 B. 6

C. 8 D. 10 E. NOTA

27. An eigenvalue λ is any number for which $\det(A-\lambda I)=0$ where A is any $n \times n$ matrix and I is the $n \times n$ identity matrix. If the complex eigenvalues of the following matrix can be represented as $\lambda_1=a_1+b_1i$ and $\lambda_2=a_2-b_2i$, where b_1 and b_2 are not equal to zero, find $\left|\dfrac{a_1+a_2}{b_1+b_2}\right|$.

$$\begin{bmatrix} 4 & 2 & 4 \\ 2 & 1 & 1 \\ -4 & -1 & -3 \end{bmatrix}$$

A. 0 B. $\dfrac{5}{4}$ C. $\dfrac{3}{2}$ D. $\dfrac{13}{7}$ E. NOTA

Use the following information for questions 28-30:

> In a circuit, the voltage over component(s) is equal to the current running through the component(s) times the impedance over the component(s). Additionally, when voltages and currents are written in the form $re^{i\theta}$, the angle θ is called the phase angle. Average power can be found with the formula $P=\dfrac{1}{2}|V||I|\cos\phi$ where V is the voltage, I is the current, and ϕ is the **difference** between the phase angles of the voltage and current.

28. What current (in amps) runs through an inductor with an impedance of $2i$ ohms if the voltage across the inductor is $8e^{i50^\circ}$ volts?

A. $4e^{-i40^\circ}$ B. $4e^{i50^\circ}$ C. $16e^{i50^\circ}$ D. $16e^{i140^\circ}$ E. NOTA

29. Find the average power (in watts) absorbed by the inductor in Problem 28.

A. 0 B. 16 C. 32 D. 64 E. NOTA

30. A current of $3e^{i25^\circ}$ amps runs through a branch with impedance $3\sqrt{3}-3i$ ohms. What is the average power (in watts) absorbed by the branch?

A. 0 B. $\dfrac{27\sqrt{3}}{2}$ C. $\dfrac{27\left(\sqrt{6}+\sqrt{2}\right)}{2}$ D. $27\sqrt{2}$ E. NOTA