Alpha Equations and Inequalities Solutions			MAO National Convention 2010		
1. B	6. B	11. B	16. B	21. B	26. A
2. A	7. B	12. A	17. A	22. D	27. B
3. B	8. D	13. B	18. C	23. D	28. C
4. C	9. B	14. B	19. E	24. A	29. A
5. C	10. C	15. A	20. D	25. B	30. E

The following were changed at the resolution center at the convention: 10 E

- 1. The solution set for the inequality is (-6, 22/3). This set contains 5+1+7=13 integers.
- 2. g(3) = 2 and f(2) = -3/2
- 3. $144 = 2^4 3^2$. Equate and solve a simple system to get x = 3, y = -1.
- 4. We have $x = \frac{f^{-1}(x)}{3f^{-1}(x) + 4}$. Solving for $f^{-1}(x)$, we get $f^{-1}(x) = \frac{4x}{1 3x}$
- 5. Cross-multiplying, we have: 2x+11 = A(x+2) + B(x-1). Setting x = 1, we see that $A = \frac{13}{3}$. Setting x = -2, we see that $A = -\frac{7}{3}$. $A B = \frac{20}{3}$
- 6. We can subtract the area of the triangle formed by $y = \frac{1}{2}x$, y = -2x + 10, and the *x*-axis from the area of the triangle formed by y = 3x, y = -2x + 10, and the *x*-axis. By finding where the lines intersect, we can easily calculate the area to be: 15 5 = 10.
- 7. Factor: $x^2 x y^2 y = (x + y)(x y 1) = 0$. The equation holds if and only if (x + y) = 0 or (x y 1) = 0. These two lines intersect at (1/2, -1/2)
- 8. Write $\frac{z+3}{z-3} 3 < 0$. Simplifying, we get $\frac{6-z}{z-3} < 0$. Checking to the left and rights of z = 3 and z = 6, we see that the solution set is $(-\infty, 3) \cup (6, \infty)$.
- 9. Vectors are perpendicular if their dot product is 0. $\langle x+3, x-3 \rangle \Box \langle x-3, x+1 \rangle = (x+3)(x-3) + (x-3)(x+1) = (x-3)(2x+4) = 0$. Sum: 3+(-2)=1.
- 10. The only real solutions are 1 and -1.
- 11. The area of the fence will be: $x \cdot (100-2x)$, where x is the length of one of the two sides not opposite the wall. This parabola achieves maximum value when x=25. At x=25, the maximum value (area) is 1250
- 12. If a and b are the roots, then a + b = 12, or $a^2 + a = 12$. So a can be 3 or -4. Then c would be 27 or -64.
- 13. Both numerator and demonimator have the same degree, so we take the ratio of the leading coefficients. -4/3
- 14. The third side length must be greater than 10 and less than 30. There are 19 such integers.
- 15. The circle is centered at the origin. The point lies outside. The shortest distance is thus the radius subtracted from the distance from (0,0) to the point. 5-2=3.

16.
$$\binom{6}{4} \left(x\sqrt{2} \right)^4 \left(\frac{\sqrt{c}}{x^2} \right)^2 = 15 \cdot 4c = 60c = 120 \cdot c = 2$$

17. $\left(x + \frac{1}{x} \right)^2 = 9 = x^2 + \frac{1}{x^2} + 2 \cdot \text{So } x^2 + \frac{1}{x^2} = 7 \cdot \left(x^2 + \frac{1}{x^2} \right) \left(x + \frac{1}{x} \right) = 21 = x^3 + \frac{1}{x^3} + x + \frac{1}{x} \cdot \text{So}$
 $x^3 + \frac{1}{x^3} = 21 - 3 = 18$. Similarly, we find $x^5 + \frac{1}{x^5} = 18 \cdot 7 - 3 = 123$.

- 18. Determinant is $1(-x) x(x^2 x^2) 1(x) = -2x$. It equals 1 only when x=-1/2.
- 19. $-y-10 < 3y+6 \Rightarrow y > -4$. $3y+6 \le 4y-2 \Rightarrow 8 \le y$. The intersection occurs when $y \ge 8$.
- 20. B = 4A and B + 8 = 2(A + 8). A=4, B=16
- 21. $\sqrt{m + \sqrt{m + \sqrt{m + \cdots}}} = 10 = \sqrt{m + 10} \Longrightarrow m = 90$.
- 22. Zeros are 1, -1, -4, 5. Testing to the left and right of each zero, we find: $[-4, -1] \cup [1, 5]$
- 23. Sum of the coefficients of f(x) is f(1), or $f(3 \cdot 2 5) = 2^4 + 2^3 3 \cdot 2 + 7 = 25$
- 24. Constant term is f(0), or $f(4 \cdot -2 + 8) = 2(-2)^3 + 2(-2)^2 3(-2) + 5 = 3$
- 25. Maximum is $\sqrt{1^2 + 2^2}$
- 26. $2^0 + 2^1 + 2^2 + \dots + 2^n = 2^{n+1} 1 = 1023$. n=9
- 27. $f(x) = \frac{1}{2}x + 2$. f(x)=5 when x=9
- 28. A square root function can only take non-negative numbers, so ln x must be non-negative. This holds for all $x \ge 1$.
- 29. We need $kx = x^2 x + 16$ to have one solution. So the discriminant must be 0. The discriminant is $(k+1)^2 - 64$. This is 0 when k=7 and -9. Sum is -2.
- 30. $1 = \sin^2 x + y^2$ can be rewritten as $\cos^2 x y^2 = 0$, or $(\cos x y)(\cos x + y) = 0$. The graph is thus the graphs $y = \cos x$ and $y = -\cos x$.