Directions: For all questions, answer choice E. "NOTA" means none of these is correct. For all questions, $i = \sqrt{-1}$ and e is the base of the natural logarithm. All variables represent real numbers.

1. Let $g(x) = 7x - 2$ Find $\dfrac{g(a) - g(b)}{a - b}$ for $a \ne b$.

     A. 7   B. $7(a - b)$ C. $\dfrac{7a - 7b - 4}{a - b}$ D. $\dfrac{7a - 7b + 2}{a - b}$ E. NOTA

2. Let $f(x) = \dfrac{x - 2}{3}$, $g(x) = 14 - 7x$, $h(x) = \pi$, and $k(x) = \pi x$.

     Which of these are strictly increasing functions of x?

     A. f and g only   B. g and h only   C. f and h only
     D. f and k only   E. NOTA

3. Let $f(x) = \dfrac{2x - 5}{3}$ What is the inverse of f?

     A. $f^{-1}(x) = \dfrac{3}{2x - 5}$ B. $f^{-1}(x) = \dfrac{3x + 5}{2}$ C. $f^{-1}(x) = 3\left(\dfrac{x}{2} + 5\right)$

     D. $f^{-1}(x) = \dfrac{x}{3} + 5$ E. NOTA

4. Let $h(x) = (x - 2)^2 - 4$. For which two functions f and g below is it true that $(g \circ f)(x) = h(x)$?

     A. $f(x) = x^2; g(x) = x - 6$ B. $f(x) = x - 4; g(x) = (x - 2)^2$
     C. $f(x) = x - 4; g(x) = x$ D. $f(x) = x - 2; g(x) x^2 - 4$ E. NOTA

5. Which quadratic function below contains the data points $(1,5), (2,9)$ and $(3,7)$?

     A. $f(x) = -3x^2 + 13x - 5$ B. $f(x) = 5x^2 + 5x - 5$
     C. $f(x) = 7x^2 + 3x - 5$ D. $f(x) = 9x^2 + x - 5$ E. NOTA

6. Which of the following functions are even?

     I. $f(x) = (x - 9)^2 + 4$ II. $g(x) = \dfrac{x^2 + 14}{5 - x^2}$ III. $h(x) = \dfrac{x - x^2}{x^2 + 5}$ IV. $k(x) = 11 + x^2$

     A. I, II, and III only   B. II and IV only   C. II, III, and IV only
     D. I, II, and IV only   E. NOTA

7. Given $f(3) = 10, f(5) = 20$, and $f(x) = \dfrac{f(x-2)}{f(x-1)}$, find $f(0)$.

   A. 2   B. 4   C. 200   D. 250   E. NOTA

8. $f(x) = 3^x - 3^{x-1} - 3^{x-2} + 3^{x-3}$ and $g(x) = k\square 3^x$ then $f(2010) = g(2007)$ for $k = ?$

   A. 27   B. 16   C. 15   D. 12   E. NOTA

9. If f and g are functions, where $f(x) = x^3 - 10x^2 + 27x - 18$ and $g(x) = x^3 - x^2 - 6x$,
   which of the following gives a relationship between f and g?

   A. $g(x) = 3f(x)$ B. $g(x) = f(x) + 3$ C. $g(x) = f(x-3)$
   D. $g(x) = f(x+3)$ E. NOTA

10. If $f(x) = x^2 + 1$ for the restricted domain and the minimum value of f is 1, then
    g is the inverse function of f. If $g(5) = -2$ then which statement must be true?

    A. $g(2) > g(1.9)$ B. g has domain $[0, \infty)$ C. $g(1) = f(1)$
    D. The range of g is $(-\infty, 0]$ E. NOTA

11. The function f, where $f(x) = (1+x)^2$, is defined for $-2 \le x \le 2$. What is the range
    of f?

    A. $0 \le f(x) \le 4$ B. $0 \le f(x) \le 9$ C. $1 \le f(x) \le 4$
    D. $1 \le f(x) \le 5$ E. NOTA

12. $p(t) = 110 + \sin(160\pi t)$

    A certain person's blood pressure, $p(t)$, in millimeters of mercury, is modeled as a function of
    time, t, in minutes. According to this model, how many times in the interval $0 \le t \le 1$ does the
    person's blood pressure reach its maximum of 111 millimeters of mercury?

    A. 80   B. 110   C. 60   D. 36   E. NOTA

13. If $f(x) = \dfrac{1}{x-5}$ and $g(x) = \sqrt{x+4}$, what is the domain of $f - g$?

    A. All x such that $x \ne 5, x \ge 4$ B. All x such that $x \ne -5, x \ge -4$
    C. All x such that $x \ne 5, x \ge -4$ D. All x such that $x \ne 5, x \le -4$
    E. NOTA

14. For which of the following functions does $f(x,y) = -f(-x,-y)$ for all values of x and y?

  A. $f(x,y) = x + y^2$ B. $f(x,y) = x - y^2$ C. $f(x,y) = x^2 - y$

  D. $f(x,y) = x + y^3$ E. NOTA

15. If $f(x) = 3\ln(x) - 1$ and $g(x) = e^x$, then $f(g(5)) =$

  A. $\dfrac{25}{e}$ B. 12   C. 14   D. $\dfrac{125}{e}$ E. NOTA

16. Which of the following satisfies the inequality $3\dfrac{x^2}{\sqrt{x}} - \dfrac{\sqrt{x+1}}{|x-2|} \leq x$?

  A. ½   B. 1   C. 3   D. 8   E. NOTE

17. What is the range of the function f, where $f(x) = -4 + \sin(2x + 5\pi)$?

  A. $-7 \leq f(x) \leq 3$ B. $-7 \leq f(x) \leq -1$ C. $-3 \leq f(x) \leq 3$

  D. $-3 \leq f(x) \leq -1$ E. NOTA

18. Determine the domain of the function $h(x) = \dfrac{3x}{x(x^2 - 16)}$.

  A. All real numbers except ±4 and 0   B. All real numbers except ±4
  C. All real numbers except ±16 and 0   D. All real numbers except 4
  E. NOTA

19. What is the amplitude of $y = 3\sin(\pi x) + 3\cos(\pi x)$?

  A. $3\sqrt{2}$ B. 3   C. 6   D. $\sqrt{6}$ E. NOTA

20. If $(a+5)(b-1) \geq 0$ then which **cannot** be true?

  A. $a \geq -5$ and $b = 2$ B. $a \leq -5$ and $b = -6$ C. $a \leq -5$ and $b = -4$

  D. $a \geq -5$ and $b = -2$ E. NOTA

21. What is the phase shift of $y = 5\cos(\pi x + 1)$?

    A. 5   B. π C. $-\dfrac{1}{2}$ D. $\dfrac{2\pi}{5}$ E. NOTA

22. How many negative integral values does the range of $f(x) = \dfrac{x(16 - x^2)}{3}$ have, for a restricted domain $[0,6]$?

    A. 1   B. 2   C. 5   D. 6   E. NOTA

23. For $y \geq 1$, $3x - 4y + 10 = 0$ is an asymptote of top half (above the x-axis) of the graph of the non-degenerate conic $4(y - 1)^2 - B(x + 2)^2 = 4B$ then which is the value of B ?

    A. $\dfrac{1}{9}$ B. $\dfrac{2}{3}$ C. $\dfrac{3}{2}$ D. $\dfrac{9}{4}$ E. NOTA

24. Which is an equation of a cubic function which has zeros $4,\ 3 + \sqrt{2}$?

    A. $f(x) = x^3 - 10x^2 + 31x - 28$ B. $f(x) = x^3 + 10x^2 + 31x - 28$
    C. $f(x) = x^3 + 10x^2 + 31x + 28$ D. $f(x) = x^3 - 10x^2 + 31x + 28$
    E. NOTA

25. For $0 < \varphi < \dfrac{\pi}{2}$ and $0 < \Omega < \dfrac{\pi}{2}$, $\tan\varphi = \dfrac{3}{4}$ and $\sin\Omega = \dfrac{7}{25}$. Find $\cos(\Omega + \varphi)$.

    A. $\dfrac{103}{100}$ B. $\dfrac{607}{750}$ C. $\dfrac{23}{27}$ D. $\dfrac{3}{5}$ E. NOTA

26. Which are the distinct roots of the polynomial $y = x^4 + 10x^3 + 36x^2 + 56x + 32$?

    A. $x = -2, 2, 4$ B. $x = 2, 4$ C. $x = -2, 4$ D. $x = -2, 2$
    E. NOTA

27. Two roots of the function $y = x^3 + 3x^2 + kx - 12$ are real and unequal but have the same absolute value. What is the value of the constant k ?

    A. 4   B. -4 C. 6   D. -9 E. NOTA

28. Find the sum of all solutions in the interval $[0, 2\pi)$ for the equation $\sec(3x) = \sqrt{2}$.

A. $\dfrac{2\pi}{3}$ B. 2π C. 6π D. 10π E. NOTA

29. What are the vertical asymptotes of $y = \dfrac{2x^2 + 11x + 5}{x^2 + 2x - 120}$?

A. $x = -5,\ x = -\dfrac{1}{2}$ B. $x = 10,\ x = -12,\ x = \dfrac{2}{3}$

C. $x = 10,\ x = -12,\ x = \dfrac{1}{3}$ D. $x = 10, x = -12$ E. NOTA

30. What is the sum of the zeros of the polynomial function $y = 2x^6 - x^5 + 3x^2 + 2x$?

A. 2   B. −2   C. $\dfrac{1}{2}$ D. $-\dfrac{1}{2}$ E. NOTA