*The acronym NOTA denotes that "none of these answers" are correct. The domain and range of functions are assumed to be $\mathbb{R}$ or the appropriate subset of $\mathbb{R}$. DNE stands for Does Not Exist. $\mathbb{R}$ denotes the real numbers and $\mathbb{N}$ denotes the natural numbers. Have fun!*

1. It is well known that $\int \frac{\sin(x)}{x}dx$ cannot be evaluated by standard integration techniques. Use the first three nonzero terms of the Maclaurin polynomial for $\sin(x)$ to approximate $\int_0^1 \frac{\sin(x)}{x}dx$.

A. $\frac{331}{720}$ B. $\frac{101}{120}$ C. $\frac{127}{144}$ D. $\frac{1703}{1800}$ E. NOTA

2. Using the ratio test, what can you conclude about $\sum_{x=1}^{\infty} \frac{(2x)!}{(x!)x^x}$?

A. Converges B. Diverges C. Inconclusive D. Converges absolutely E. NOTA

3. Find the area bound by the polar curve $r^2 = \frac{25}{4+5\sin^2(\theta)}$.

A. $\frac{25\pi}{12}$ B. $\frac{5\pi\sqrt{2}}{2}$ C. $\frac{25\pi}{6}$ D. $5\pi\sqrt{2}$ E. NOTA

4. Let $S$ be the set of all ellipses for which the sum of the semimajor and semiminor axes is 7. Find the average area of an ellipse in $S$. (Hint: $A = \pi ab$)

A. $\frac{7\pi}{4}$ B. $\frac{49\pi}{6}$ C. $\frac{49\pi}{4}$ D. $\frac{28\pi}{3}$ E. NOTA

5. Let $\vec{u} =< 1, t+2, 2t^2-3 >$ and let $\vec{v} =< -4, t, 2t >$. Find the minimum possible value of the dot product of $\vec{u}$ and $\vec{v}$.

A. $-\frac{21}{4}$ B. $-\frac{136}{27}$ C. $-\frac{9}{4}$ D. $-\frac{56}{27}$ E. NOTA

6. Consider the polar cardioid $r = 2 - 2\sin(\theta)$. Find the slope of the tangent line at $\theta = \frac{\pi}{4}$.

A. $\sqrt{2}-2$ B. $1-\sqrt{2}$ C. $\sqrt{2}-1$ D. $2-\sqrt{2}$ E. NOTA

7. Compute $\lim_{t\to\infty} e^{-t}\ln(e^t+1)$.

A. 0 B. $\frac{1}{e}$ C. 1 D. $\infty$ E. NOTA

8. Let $f(x) = \ln(x)$. Which expression is equivalent to $\frac{1}{f'(x)} + \frac{1}{f''(x)} + \frac{1}{f'''(x)} + ...$?

A. $\ln(x+1)$ B. $x\ln(x+1)$ C. $e^{-x}$ D. $xe^{-x}$ E. NOTA

9. Evaluate: $\int_{e^2}^{e^4} \frac{\ln(x)}{\sqrt{x}}dx$.

A. $e^2$ B. $2e^2$ C. $4e^2+4$ D. $12e^2+8e$ E. NOTA

10. Evaluate: $\int_0^{\frac{\pi}{6}} \sqrt{\frac{1+\sin(x)}{1-\sin(x)}}dx$

A. $\ln 2 - \frac{1}{2}\ln 3$ B. $\frac{1}{2}\ln 3$ C. $\ln 2$ D. $\ln 2 + \frac{1}{2}\ln 3$ E. NOTA

11. Let $f$ be a differentiable even function equal to its Maclaurin series. Which of the following statements are true?

I. The coefficient of the $x^3$ term in the Maclaurin series for $f$ is zero

II. $\int_a^b f(x)dx = \int_{-b}^{-a} f(x)dx$

III. The derivative of $f$ is an odd function

A. I, II only B. I, III only C. II, III only D. I, II, III E. NOTA

12. Find the area of one petal of the polar rose $r = 2\cos(3\theta)$. (Hint: it has 3 petals)

A. $\frac{\pi}{3}$ B. $\frac{\pi}{3} + \frac{\sqrt{3}}{12}$ C. $\frac{2\pi}{3}$ D. $\frac{\pi}{3} + \frac{\sqrt{3}}{12}$ E. NOTA

For questions 13 and 14, consider the parametric curve given by the equations $y = \sin(t), x = \tan(t), 0 < t < \frac{\pi}{3}$ (i.e. $0 < x < \sqrt{3}$).

13. Find the area of the region bound by $x = 0$, $x = \sqrt{3}$, the $x$-axis, and this curve.

A. $\frac{\sqrt{3}}{2}$ B. 1 C. $\sqrt{3}$ D. 2 E. NOTA

14. The portion of this curve between $x = 0$ and $x = \sqrt{3}$ is rotated about the $x$-axis, forming a solid. What is its volume?

A. $\frac{4\pi^2 - 3\pi\sqrt{3}}{24}$ B. $\frac{3\pi\sqrt{3} - \pi^2}{3}$ C. $\frac{7\pi}{3}$ D. $2\pi\sqrt{3}$ E. NOTA

15. Consider the differential equation $\sqrt{1-x^2}\frac{dy}{dx} - y^2 = 1$. If $y(0) = 0$, what is $y(\frac{1}{2})$?

A. $\frac{\sqrt{3}}{3}$ B. $\frac{\sqrt{2}}{2}$ C. $\frac{\sqrt{3}}{2}$ D. $\sqrt{3}$ E. NOTA

16. Evaluate: $\int_0^2 \frac{1}{x^2 - 1}dx$

A. $\frac{1}{2}\ln(2)$ B. $\frac{1}{2}\ln(3)$ C. $\ln(2)$ D. $\ln(3)$ E. NOTA

17. Consider the triangle with vertices at the points $(0, 0)$, $(a, 0)$, and $(0, b)$, where $a = \frac{1}{b\sqrt{b}} > 0$. If this triangle is rotated about the line $y = -x$ (on which the point (0, 0) lies), what value of $b$ minimizes the volume of the solid formed? (Hint: the centroid of the triangle is $(\frac{a}{3}, \frac{b}{3})$ )

A. 1 B. $\sqrt[5]{2}$ C. $\sqrt[3]{2}$ D. $\sqrt[5]{4}$ E. NOTA

18. Consider the parametric equations $y = \ln(t)$, $x = t^2 + 1$. Compute $\frac{d^2y}{dx^2}$.

A. $-\frac{1}{2t^4}$ B. $-\frac{1}{2t^3}$ C. $-\frac{1}{t^3}$ D. $\frac{2}{t}$ E. NOTA

19. Evaluate $2\pi \int_0^\infty re^{-r^2}dr$.

A. $\frac{\pi}{2}$ B. $\pi$ C. $2\pi$ D. $4\pi$ E. NOTA

20. Evaluate: $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{2}{n}\left(\frac{(2k)^4}{n^4}\right)$

A. 4 B. $\frac{32}{5}$ C. $\frac{32}{3}$ D. 16 E. NOTA

21. Evaluate: $\int_0^{\frac{1}{\sqrt{2}}} \frac{4x}{1-x^4}dx$

A. $\ln(4) - \ln(3)$ B. $\ln(2)$ C. $\ln(3)$ D. $\ln(4)$ E. NOTA

22. Evaluate: $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + ...$ when $x = 2$.

A. $\ln(3)$ B. $\arctan(2)$ C. $2\ln(2)$ D. 4 E. NOTA

23. Find the radius of convergence of the series $\sum_{k=1}^{\infty} \frac{(kx)^k}{k!}$

A. 0 B. $\frac{1}{e}$ C. $e$ D. $\infty$ E. NOTA

24. The factorial operation can be extended from $\mathbb{N}$ to $\mathbb{R}$ by defining $t! = \int_0^{\infty} x^t e^{-x} dx$ (note that under this definition, we still get $0! = 1, 1! = 1, 2! = 2, 3! = 6, 4! = 24$, etc). Evaluate $(\frac{1}{2})!$ (or, in other words, $\int_0^{\infty} x^{\frac{1}{2}} e^{-x} dx$). Hint: use the fact that $\int_{-\infty}^{\infty} e^{-t^2} dt = \sqrt{\pi}$.

A. $\frac{\sqrt{\pi}}{2\sqrt{2}}$ B. $\frac{\sqrt{\pi}}{2}$ C. $\frac{\sqrt{\pi}}{\sqrt{2}}$ D. $\sqrt{\pi}$ E. NOTA

25. Let $\Theta(x) = 1 + x + x^2 + ....$ Evaluate $\int_0^{0.5} \Theta(x)dx$.

A. $\ln(2)$ B. $\ln(4)$ C. $\ln(8)$ D. $\ln(16)$ E. NOTA

26. What shape is the polar graph of $r = 2010\sin(\theta)$?

A. Line B. Cardioid C. Lemniscate D. Circle E. NOTA

27. If $\frac{1}{x^3 - 3x^2 + 2x}$ is written as $\frac{\mu}{x} + \frac{\alpha}{x-2} + \frac{\theta}{x-1}$, what is $\mu\alpha\theta$?

A. $-\frac{1}{4}$ B. $-\frac{1}{8}$ C. $\frac{1}{27}$ D. $\frac{1}{4}$ E. NOTA

28. The region bound above by $y = \frac{1}{x^{\frac{3}{4}}}$, below by $y = 0$, and to the right of $x = 1$ is revolved about the $x$-axis, creating a solid. Find its volume.

A. $\frac{2\pi}{3}$ B. $\pi$ C. $2\pi$ D. $3\pi$ E. NOTA

29. Let $y = \int_0^3 3x^2 dx$. What is the average value of $y$ on $[0, 3]$?

A. $\frac{9}{2}$ B. $\frac{27}{4}$ C. 9 D. $\frac{27}{2}$ E. NOTA

30. Evaluate: $\int_1^2 \frac{1}{\sqrt{4-x^2}}dx$

A. $\frac{\pi}{6}$ B. $\frac{\pi}{4}$ C. $\frac{\pi}{3}$ D. $\frac{\pi}{2}$ E. NOTA