

For all questions, answer E. "NOTA" means none of the above answers is correct."

1. Evaluate $\lim_{x \rightarrow \infty} (3x - 2) \ln \left(\frac{3x - 1}{3x - 2} \right)$.

- A) 0 B) 1 C) 2 D) 3 E) NOTA

2. Use differentials to approximate the value of $\sin(60^\circ (3600 / \pi)')$.

- A) $\frac{2\sqrt{3}-1}{6}$ B) $\frac{3\sqrt{3}+1}{6}$ C) $\frac{\sqrt{3}+1}{3}$ D) $\frac{3\sqrt{3}-1}{2}$ E) NOTA

3. For what values of x will \sqrt{x} approximate $\sqrt{x+1}$ with an allowable error less than or equal to .001.

- A) $0 \leq x < 75000$ B) $75000 \leq x < 150000$ C) $150000 \leq x < 225000$

- D) $x \geq 250000$ E) NOTA

4. Evaluate $\lim_{x \rightarrow \infty} \frac{6x + 2 \ln x}{x + 3 \ln x}$.

- A) 0 B) 3 C) 6 D) DNE E) NOTA

5. Given that $F_1 = 1$ and $F_2 = 1$ and $F_n = F_{n-1} + F_{n-2}$ find $\lim_{n \rightarrow \infty} \frac{F_{n+2}}{F_n}$.

- A) $\frac{1+\sqrt{5}}{2}$ B) $\frac{2+\sqrt{5}}{2}$ C) $\frac{3+\sqrt{5}}{2}$ D) $\frac{4+\sqrt{5}}{2}$ E) NOTA

6. Find $f' \left(\frac{\pi}{3} \right)$ where $f(x) = 1 + \cos(x) + \cos^2(x) + \cos^3(x) + \dots$.

- A) $-2\sqrt{3}$ B) $-\frac{2\sqrt{3}}{3}$ C) $\frac{2\sqrt{3}}{3}$ D) $2\sqrt{3}$ E) NOTA

7. How many of the following expressions are of indeterminate form?

- I) 0^0 II) ∞^{0^∞} III) $\infty - \infty$ IV) 1^∞

- A) 1 B) 2 C) 3 D) 4 E) NOTA

8. A tangent line and a normal line are drawn on the graph of $f(x) = e^x$ at $x = \frac{\pi}{2}$ and form a

triangle with the x -axis. If A represents the length of the side formed by the tangent line, B represents the length of the side formed by the normal line, and C represents the length

of the side formed by the x -axis, find $\frac{A^2 + B^2 - C}{e^\pi}$.

- A) $e^\pi - 1$ B) $e^\pi + 1$ C) $e^{2\pi} - 1$ D) $e^{2\pi} + 1$ E) NOTA

9. How many of the following functions are differentiable on their entire domain?

I) $h(x) = \left| (x-4)^{3/2} \right|$ II) $g(x) = \sin x$ III) $k(x) = \ln \left| x^3 - \frac{1}{2} \right|$ IV) $j(x) = \cos |x|$

- A) 1 B) 2 C) 3 D) 4 E) NOTA

10. Given that $x^3y^2 + x^2y \cos(y^2) = 0$ and $x \neq 0$ find $\frac{dx}{dy}$ when $y = \sqrt{\pi}$.

A) $\frac{2\sqrt{\pi}-2}{\pi}$ B) $\frac{2\sqrt{\pi}+2}{\pi}$ C) $\frac{2\pi-1}{\pi^2}$ D) $\frac{2\pi+1}{\pi^2}$ E) NOTA

11. An airplane is flying at a constant height of 12 miles on a path that will take it directly over a radar tracking station. If the distance between the airplane and the station in miles, S , is decreasing at a rate of 400 miles per hour when $S = 20$, what is the speed of the plane in miles per hour?

- A) 2000 B) 1000 C) 500 D) 250 E) NOTA

12. A parametric equation is given such that $x = \sin t$ and $y = \cos t$. Find $\frac{d^2y}{dx^2}$ when $t = \pi$.

A) $\frac{3}{2}$ B) 1 C) $\frac{1}{2}$ D) 0 E) NOTA

13. Find the guaranteed value x_0 as prescribed by the Mean Value Theorem for Derivatives, given $f(x) = 3x^2 - 1$ on the interval $[1,4]$.

A) 3 B) $\frac{5}{2}$ C) 2 D) $\frac{3}{2}$ E) NOTA

14. Use the tangent line approximation of $f(x) = \cos x + \sin x + 5$ at $x = 0$ to approximate the value of $\cos(-0.5) + \sin(-0.5) + 5$.

A) $\frac{13}{2}$ B) $\frac{9}{2}$ C) $\frac{5}{2}$ D) $\frac{1}{2}$ E) NOTA

15. A given periodic function is defined such that $k(x) = \cos(2010x) + \sin(1005x)$. At what value of x does the minimum value of $y = Ax^2 + Bx + C$ occur if A = the numerator of the period of $k(x)$, B = the denominator of the period of $k(x)$, and C = the period of $k(x)$.

A) 0 B) $-\frac{2010}{\pi}$ C) $-\frac{1005}{2\pi}$ D) $-\frac{1005}{4\pi}$ E) NOTA

16. Given that $f(x) = 3x^3 \cos x$ evaluate $\lim_{h \rightarrow 0} \frac{f(x-3h) + f(x+2h) - f(x+3h) - f(x-4h)}{h}$.

- A) $9x^2 \cos x \sin x$ B) $3x \cos x + 18x^2 \sin x \cos x$ C) $9 \cos x$ D) 0 E) NOTA

17. Evaluate $\lim_{x \rightarrow 0^+} \frac{\cot x}{\cot 2x}$ given that x is in degrees.

- A) 0 B) 1 C) 2 D) $\frac{\pi}{90}$ E) NOTA

18. Find $f^{2010}(x)$ given that $f(x) = xe^x - 2010e^x$.

- A) xe^x B) $xe^x - e^x$ C) $2010e^x$ D) $2010xe^x$ E) NOTA

19. Given that $a_n = \sqrt{20 - a_{n-1}}$ and $a_1 = 20$, find $\lim_{n \rightarrow \infty} a_n$.

- A) $\sqrt{20}$ B) 4 C) $\sqrt{5}$ D) 1 E) NOTA

20. For $f(x) = \sqrt{x - \sqrt{x - \sqrt{x - \sqrt{x \dots}}}}$, let $g(x) = f^{-1}(x)$. Find $g'(1)$.

- A) $\sqrt{7}$ B) $\sqrt{6}$ C) $\sqrt{5}$ D) 2 E) NOTA

21. Let A = the number of critical values of $f(x) = \frac{x^2 - 5x + 1}{x - 3}$ and let B = the number of

points of inflection of $g(x) = \frac{x^2}{x + 4}$. Find the product AB .

- A) 0 B) 1 C) 2 D) 3 E) NOTA

22. Use two iterations of Newton's method, beginning with $x_1 = 0$, to approximate a root of $x^3 + 4x - 5$.

- A) $\frac{45}{82}$ B) $\frac{346}{255}$ C) $\frac{298}{235}$ D) $\frac{285}{278}$ E) NOTA

23. Evaluate $\lim_{x \rightarrow \infty} x^{(1/x)}$.

- A) -1 B) 0 C) 1 D) DNE E) NOTA

24. Use differentials to approximate the value of $\sqrt[3]{63}$ given that $\sqrt[3]{64} = 4$.

- A) $\frac{19}{4}$ B) $\frac{191}{48}$ C) $\frac{233}{16}$ D) $\frac{216}{13}$ E) NOTA

25. Find $\frac{dy}{dx}$ at $x = 1$ if $y = x^2 - x^y$.

- A) $\ln 2$ B) 0 C) 2 D) $\ln 5$ E) NOTA

26. Evaluate $\lim_{x \rightarrow 0} \frac{\sin x}{|x|}$.

- A) 0 B) 1 C) 2 D) DNE E) NOTA

27. Evaluate $\lim_{k \rightarrow 0^+} \frac{\sqrt{k}}{\sqrt{16 + \sqrt{k}} - 4}$.

- A) 0 B) 4 C) 8 D) DNE E) NOTA

28. Find the second derivative of $f(x) = \frac{\cos(x) - x^2}{x}$.

- A) $\frac{(2 - x^2)\cos x + 2x \sin x}{x^3}$ B) $\frac{(3 + x^2)\cos x + 2 \sin x}{x^3}$
 C) $\frac{(2 - x^2)\sin x + 2x \cos x}{x^4}$ D) $\frac{(2 - x^3)\cos x + x \sin x}{x^3}$ E) NOTA

29. For what value of k will $Q(x) = \begin{cases} x^2 - 2x(k+1) + 2, & x \geq 1 \\ x - 1, & x < 1 \end{cases}$ be both continuous and differentiable?

- A) 0 B) $-\frac{1}{2}$ C) $\frac{1}{2}$ D) No such value exists E) NOTA

30. Find $\frac{dy}{dx} \left(\int_{x^2}^{\sin x} \sqrt{1 - x^2} dx \right)$.

- A) $\cos^2 x - 2x\sqrt{1 - x^4}$ B) $\sin x \cos x - \sqrt{1 - x^2}$ C) $\cos 2x - \sqrt{1 - x^4}$
 D) $\sqrt{1 - x^2} + \sin^2 x$ E) NOTA