For all questions, answer E. "NOTA" means none of the above answers is correct. For a matrix **A**, let a_{ij} denote the entry in the *i*-th row and *j*-th column of **A**. Let $\mathbf{I}_n$ denote the $n \times n$ identity matrix and let $\mathbf{0}_{m\times n}$ denote the $m \times n$ matrix of all zeros.

1. Evaluate: $\begin{bmatrix} 7 & -2 \\ 9 & 1 \end{bmatrix} + \begin{bmatrix} -3 & 8 \\ 2 & 0 \end{bmatrix}$

A. $\begin{bmatrix} 4 & 6 \\ 11 & 1 \end{bmatrix}$ B. $\begin{bmatrix} -25 & 56 \\ -25 & 72 \end{bmatrix}$ C. $\begin{bmatrix} 10 & 10 \\ 11 & 1 \end{bmatrix}$ D. $\begin{bmatrix} 5 & 5 \\ 10 & 2 \end{bmatrix}$ E. NOTA

2. Evaluate: $\langle 1,-1,1\rangle \cdot \langle 2,5,-2\rangle$

A. –5 B. –1 C. 5 D. 12 E. NOTA

3. Let **A** be a nonsingular 4×4 matrix and let **B** be the matrix obtained when every entry of *A* is multiplied by 2. What is $\dfrac{|\mathbf{B}|}{|\mathbf{A}|}$?

A. 1 B. 2 C. 4 D. 16 E. NOTA

4. Let $\mathbf{E}_{ij}$ be the $n \times n$ matrix where the entry in the *i*-th row and *j*-th column is 1, and all other entries are 0, where $1 \le i, j \le n$. If $n \ge 5$, then what is $\mathbf{E}_{23} \cdot \mathbf{E}_{35}$?

A. $\mathbf{E}_{25}$ B. $\mathbf{E}_{32}$ C. $\mathbf{E}_{33}$ D. $\mathbf{E}_{52}$ E. NOTA

5. Evaluate: $\begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{vmatrix}$

A. –2 B. –1 C. 0 D. 1 E. NOTA

6. Let *R* be the region in the *x*-*y* plane bounded by the curves $y = x^2 + 3$ and $y = 2x + 6$. *R* is transformed to the region R' in the *u*-*v* plane by the transformation $u = 4x + 2y$ and $v = x + 2y$. What is the area of R'?

A. $\dfrac{16}{9}$ B. $\dfrac{32}{3}$ C. 32 D. 64 E. NOTA

7. Which of the following pairs of vectors in $\mathbb{R}^3$ are linearly independent?

A. $\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix}$

B. $\begin{pmatrix} 3 \\ 5 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 6 \\ 10 \\ 3 \end{pmatrix}$

C. $\begin{pmatrix} -3 \\ 6 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -2 \\ -1/3 \end{pmatrix}$

D. $\begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ 3 \\ 3 \end{pmatrix}$

E. NOTA

8. Let $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ and let $\mathbf{A}$ be a symmetric matrix such that $\mathbf{x}^T\mathbf{A}\mathbf{x} = 7x_1^2 - 4x_1x_2 + 5x_2^2$. What is the a_{12} entry of $\mathbf{A}$?

A. -4 B. -2 C. 5 D. 7 E. NOTA

9. Let $f(x)$ be a cubic polynomial with real coefficients, and let *r*, *s*, and *t* be the roots of $f(x)$. Let $\delta = \begin{vmatrix} 1 & r & r^2 \\ 1 & s & s^2 \\ 1 & t & t^2 \end{vmatrix}$. Suppose that exactly one root of $f(x)$ is real. Which of the following must be true?

A. δ^2 is a non-real complex number
B. δ^2 is a negative real number
C. δ^2 is zero
D. δ^2 is a positive real number
E. NOTA

10. Let $\mathbf{A}$ be an $n \times n$ matrix whose entries are integers. If $\mathbf{A}$ is invertible and if all the entries of $\mathbf{A}^{-1}$ are integers, then which of the following represents the set of all possible values of $|\mathbf{A}|$?

A. $\{1\}$ B. $\{-1,1\}$ C. $\{-1,0,1\}$ D. $\mathbb{Z}$ E. NOTA

11. Let $\mathbf{v} = \frac{1}{3}\mathbf{i} + a\mathbf{j}$, where *a* is a positive real number. For what value of *a* is $\|\mathbf{v}\| = 1$?

A. $\frac{1}{3}$ B. $\frac{\sqrt{2}}{2}$ C. $\frac{2\sqrt{2}}{3}$ D. $\frac{8}{9}$ E. NOTA

For questions 12, 13, and 14 let $\mathbf{A} = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 6 & -3 \\ -2 & -4 & 2 \end{bmatrix}$.

12. What is the trace of **A**?

A. 0 B. 3 C. 6 D. 9 E. NOTA

13. What is the rank of **A**?

A. 0 B. 1 C. 2 D. 3 E. NOTA

14. The null space of a matrix **A** is the set of all vectors **x** such that $\mathbf{Ax} = \mathbf{0}_{3\times 1}$. Which of the following vectors is **not** in the null space of **A**?

A. $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ B. $\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$ C. $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ D. $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ E. NOTA

15. Let **A** be a 3×4 matrix and let **B** be a 3×2 matrix. Which of the following products is defined?

A. $\mathbf{BA}$ B. $\mathbf{B}^T\mathbf{A}$ C. $\mathbf{BA}^T$ D. $\mathbf{B}^T\mathbf{A}^T$ E. NOTA

16. Let **A** be an $n\times n$ matrix with real entries, and let **B** be the $n\times n$ matrix such that $b_{ij} = \sum_{k=1}^{n} a_{ik}a_{jk}$. If $|\mathbf{A}| = 4$, then what is the value of $|\mathbf{B}|$?

A. 0 B. 4 C. 8 D. 16 E. NOTA

17. Let **A** be an $n\times n$ singular matrix. How many of the following statements must be true?

I. $\det(\mathbf{A}) = 0$
II. There exists a real non-zero *n*-dimensional vector **x** such that $\mathbf{Ax} = \mathbf{0}_{n\times 1}$
III. $\mathbf{A}^T$ is singular
IV. The trace of **A** is 0

A. 1 B. 2 C. 3 D. 4 E. NOTA

18. Let θ be the acute angle (measured in radians) between the vectors $\langle 1,1\rangle$ and $\langle 1+t, 1-t\rangle$ after *t* seconds, where $t \geq 0$. What is $\frac{d\theta}{dt}$ when $t = 1$?

A. $-\frac{\sqrt{2}}{4}$ B. 0 C. $\frac{1}{2}$ D. $\frac{\sqrt{2}}{2}$ E. NOTA

19. Let $D(x)=\begin{vmatrix} x & 1 \\ e^x & x+1 \end{vmatrix}$. Find $D'(0)$.

A. 1 B. 2 C. 3 D. 4 E. NOTA

20. Let $\mathbf{A}=\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$. Find the maximum value of $\|\mathbf{Ax}\|$ where $\mathbf{x}$ is a 2×1 vector such that $\|\mathbf{x}\|=1$.

A. $\frac{1}{2}$ B. 1 C. $\frac{3}{2}$ D. 2 E. NOTA

21. Let $\mathbf{A}$ be an $n\times n$ matrix such that $\mathbf{A}^2=\mathbf{A}+2\mathbf{I}_n$. Which of the following is equal to $\mathbf{A}^3$?

A. $\mathbf{A}+2\mathbf{I}_n$ B. $2\mathbf{A}+2\mathbf{I}_n$ C. $3\mathbf{A}+2\mathbf{I}_n$ D. $4\mathbf{A}+2\mathbf{I}_n$ E. NOTA

22. Let $\mathbf{v}=\mathbf{i}+x\mathbf{j}+y\mathbf{k}$. If $\|\mathbf{v}\times\mathbf{i}\|=1$, then what is $\|\mathbf{v}\|$?

A. 1 B. $\sqrt{2}$ C. $\sqrt{3}$ D. 2 E. NOTA

23. How many solutions does the following system of equations have?

$$2x-6y=8$$
$$-3x+9y=-12$$

A. 0 B. 1 C. 2 D. Infinitely many E. NOTA

24. Let $\mathbf{v}$ be the vector $\langle t, 2+t\rangle$, where $t\in\mathbb{R}$. For what value of t is $\|\mathbf{v}\|$ minimized?

A. -2 B. $-\frac{1}{2}$ C. 0 D. $\frac{1}{2}$ E. NOTA

25. Let $\mathbf{u}$ and $\mathbf{v}$ be vectors in the x-y plane. For which of the following pairs of vectors will $\mathbf{u}\times\mathbf{v}=\langle 0,0,a\rangle$, where a is a positive real number?

A. $\mathbf{u}=\langle 5,1,0\rangle$ and $\mathbf{v}=\langle 2,3,0\rangle$ B. $\mathbf{u}=\langle 1,3,0\rangle$ and $\mathbf{v}=\langle 2,1,0\rangle$
C. $\mathbf{u}=\langle -1,1,0\rangle$ and $\mathbf{v}=\langle 1,1,0\rangle$ D. $\mathbf{u}=\langle -1,-2,0\rangle$ and $\mathbf{v}=\langle -2,2,0\rangle$ E. NOTA

26. Alex and Brian go to the grocery store to purchase apples and oranges. Alex purchases four apples and seven oranges for a total of \$43. Brian purchases three apples and one orange for a total of \$11. How much does one orange cost?

A. \$2 B. \$3 C. \$4 D. \$5 E. NOTA

27. For which of the following values of λ does there exist a nonzero vector **x** such that $\begin{bmatrix} -1 & 2 \\ 8 & -1 \end{bmatrix}\mathbf{x} = \lambda\mathbf{x}$?

A. 1 B. 2 C. 3 D. 4 E. NOTA

28. Let $\mathbf{A} = \begin{bmatrix} 1 & 0 & -1 \\ 7 & 2 & -3 \\ 2 & 3 & 5 \end{bmatrix}$. What is the cofactor of a_{23}?

A. -9 B. -3 C. 3 D. 9 E. NOTA

29. Let **A** be an 11×11 matrix such that $\mathbf{A}^T = -\mathbf{A}$. Which of the following is true?

A. $|\mathbf{A}| = 0$ B. $|\mathbf{A}| > 0$

C. $|\mathbf{A}| < 0$ D. The sign of $|\mathbf{A}|$ cannot be determined E. NOTA

30. Evaluate: $\left\|\langle 1,2,3\rangle - 2\cdot\langle -1,4,3\rangle\right\|$

A. $2\sqrt{2}$ B. 4 C. $3\sqrt{6}$ D. $\sqrt{14}+2\sqrt{26}$ E. NOTA