There were 8 changes to the key to this test made at the resolution center during the convention. The solutions page is a PDF so changes could not be made there. Please see the errata sheet in this test file.

For all questions, answer E. "NOTA" means none of the above answers is correct.

1. How many of the following are true regarding the monotonically decreasing infinite sequence $\{S_n\}$ and its associated series $\sum S_n$?

I) If $\{S_n\}$ converges then $\sum S_n$ converges.

II) Convergence (Divergence) of $\sum S_n$ is unaffected by the removal of the first *n* terms from $\{S_n\}$.

III) If $\lim_{n\to\infty} S_n = 0$ then $\sum S_n$ converges.

IV) If $\lim_{n\to\infty} S_n = \frac{3}{4}$ then $\lim_{n\to\infty} \frac{1}{\sum S_n} = 0$.

A) 1 B) 2 C) 3 D) 4 E) NOTA

2. Approximate the value of $1.2^{-1/3}$ using the first three terms (second degree) of the binomial expansion of $(1+x)^{-1/3}$.

A) $\frac{246}{225}$ B) $\frac{238}{225}$ C) $\frac{223}{225}$ D) $\frac{212}{225}$ E) NOTA

3. Evaluate $\prod_{n=2}^{\infty} \sqrt{\left(1-\frac{1}{n^2}\right)}$

A) 1 B) $\frac{\sqrt{2}}{2}$ C) $\sqrt{2}$ D) $\sqrt{3}$ E) NOTA

For questions 4-8 determine the convergence (divergence) of the given series.

4. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{2n+1}}$

A) Converges Absolutely B) Converges Conditionally C) Diverges
D) Inconclusive E) NOTA

5. $\sum_{n=1}^{\infty} \frac{\ln(n)}{n!}$

A) Converges Absolutely B) Converges Conditionally C) Diverges
D) Inconclusive E) NOTA

6. $\sum_{n=1}^{\infty}(-1)^{n+2}\left(\frac{n!}{4^n}\right)$

A) Converges Absolutely B) Converges Conditionally C) Diverges
D) Inconclusive E) NOTA

7. $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}n^2}{n^3+1}$

A) Converges Absolutely B) Converges Conditionally C) Diverges
D) Inconclusive E) NOTA

8. $\sum_{n=1}^{\infty}\frac{\ln n}{\sqrt{n}}$

A) Converges Absolutely B) Converges Conditionally C) Diverges
D) Inconclusive E) NOTA

9. Evaluate $\ln\left(\sum_{n=0}^{\infty}\frac{n^2}{n!}\right)$

A) 1 B) $\ln 2$ C) $\ln(2e)$ D) $2+\ln 2$ E) NOTA

10. Given that $R=\sum_{i=0}^{\infty}\left(\frac{1}{2}\right)^i$ and $P=\sum_{i=1}^{\infty}(-1)^{i+1}\left(\frac{1}{2i-1}\right)$, let S be an infinite series such that $S=R+P$. Assuming that the terms of S alternate successively between the terms of R and P $\left(1+1+\frac{1}{2}-\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\ldots\right)$, determine the error when S is approximated by its first 100 terms.

A) $\left(\frac{1}{2}\right)^{49}+\frac{1}{99}$ B) $\left(\frac{1}{2}\right)^{50}+\frac{1}{100}$ C) $\left(\frac{1}{2}\right)^{99}+\frac{1}{197}$ D) $\left(\frac{1}{2}\right)^{100}+\frac{1}{199}$

E) NOTA

11. Let $f(x)=\sum_{n=1}^{\infty}(-1)^{n+1}\left(\frac{(\sin x)^{2n-1}}{2n-1}\right)$. Find $f'\left(\frac{\pi}{4}\right)$.

A) $\frac{\sqrt{2}}{2}$ B) $\frac{\sqrt{2}}{3}$ C) $\frac{\sqrt{2}}{4}$ D) $\frac{\sqrt{2}}{5}$ E) NOTA

12. Given that $f(x)=x^2-6x+7$ find $\int_3^{3.2} f(x)dx$. (Hint: expand $f(x)$ in powers of $(x-3)$.)

A) $-\frac{146}{225}$ B) $-\frac{208}{225}$ C) $-\frac{149}{375}$ D) $-\frac{262}{375}$ E) NOTA

13. Given that $i=\sqrt{-1}$ evaluate the infinite series: $1-\pi i-\frac{\pi^2}{2}+\frac{\pi^3 i}{6}+\frac{\pi^4}{24}-\frac{\pi^5 i}{120}+\ldots$

A) -1 B) 0 C) 1 D) 2 E) NOTA

14. Evaluate $\frac{1}{\pi}-\frac{1}{3\pi}+\frac{1}{5\pi}-\frac{1}{7\pi}+\ldots$

A) 1 B) $\frac{1}{2}$ C) $\frac{1}{4}$ D) 0 E) NOTA

15. Determine the interval of convergence of $\sum_{n=0}^{\infty}\frac{n(x+3)^n}{(x-1)^{2n}e^{n^2}}$.

A) $(0,e^5]$ B) $(-\infty,\infty)$ C) $(-\infty,1)\cap(1,\infty)$ D) $(-\infty,1)\cap(3,\infty)$ E) NOTA

16. Evaluate $\sum_{j=0}^{\infty}\frac{1}{9j^2-3j-2}$

A) $-\frac{1}{6}$ B) $-\frac{1}{2}$ C) $\frac{1}{2}$ D) $\frac{1}{6}$ E) NOTA

17. Given that $N=\sum_{k=1}^{\infty}\frac{1}{k^2}$ evaluate $\sum_{j=1}^{\frac{\pi^2}{N}}\left(j\prod_{i=1}^{8}\tan(10i)\right)$. (Hint: N is six times larger than the product of the zeros of $y=x^2-\frac{\pi}{3}x+\frac{\pi^2}{36}$.)

A) 18 B) 19 C) 20 D) 21 E) NOTA

18. Find $0.\overline{481}_9$ as a fully reduced base 10 fraction.

A) $\frac{33}{56}$ B) $\frac{5}{8}$ C) $\frac{37}{56}$ D) $\frac{39}{56}$ E) NOTA

19. Evaluate $\lim_{n\to\infty}\sum_{i=0}^{n}\left(\frac{i}{n}\right)^{1.5}$

A) 0 B) $\frac{1}{3}$ C) $\frac{2}{3}$ D) 1 E) NOTA

20. If $R_n=\frac{1}{2}\left(a^n+b^n\right)$, where $a=3+2\sqrt{2}$, $b=3-2\sqrt{2}$, and $n=0,1,2,\ldots,$ then determine R_2 in base 2 and find the sum of its digits, also in base 2.

A) 1 B) 10_2 C) 11_2 D) 100_2 E) NOTA

21. Evaluate the following product: $\left(\sum_{n=1}^{\infty}\frac{(\pi/2)^{2n-1}}{(2n-1)!}\right)\left(\sum_{n=0}^{\infty}\frac{(-2)^n}{n!}\right)\left(\sum_{n=0}^{\infty}\frac{4^n}{n!}\right)$.

A) 0 B) e^2 C) $\frac{e\sqrt{2}}{2}$ D) $\frac{e\sqrt{3}}{2}$ E) NOTA

22. Find a power series, centered at $x=0$, for $f(x)=\frac{3x-1}{x^2-1}$.

A) $\sum_{n=0}^{\infty}3^n x^{2n}$ B) $\sum_{n=0}^{\infty}\left[2(-1)^n-1\right]x^n$ C) $\sum_{n=0}^{\infty}n!x^{n/2}$ D) $\sum_{n=0}^{\infty}\left[(-2)^n+1\right]x^n$ E) NOTA

23. Determine the radius of convergence for $\sum_{n=0}^{\infty}n!x^n$.

A) 0 B) 1 C) e D) ∞ E) NOTA

24. Evaluate $\sum_{n=0}^{\infty}\frac{1}{(n+2)n!}$. (Hint: Integrate the Maclaurin series for $f(x)=xe^x$ term-by-term over the closed interval $[0,1]$.)

A) 0 B) 1 C) 2 D) 3 E) NOTA

25. Determine the common ratio of a geometric series with first term $i+1$ and sum $3i+4$.

A) $\frac{-i+16}{25}$ B) $\frac{2i+15}{25}$ C) $\frac{i+18}{25}$ D) $\frac{-2i+13}{25}$ E) NOTA

26. The 5th term of an arithmetic sequence is 56 and the 25th term is 105. Find the 21st term.

A) $\frac{475}{3}$ B $\frac{475}{12}$ C) $\frac{515}{4}$ D) $\frac{515}{8}$ E) NOTA

27. Evaluate the limit of the following sequence: $\sqrt{2}, \sqrt{2\sqrt{2}}, \sqrt{2\sqrt{2\sqrt{2}}}, \ldots$

A) 0 B) 1 C) 2 D) 3 E) NOTA

28. Find the sum of the squares of the first 10 positive odd integers.

A) 1046 B) 1108 C) 1224 D) 1330 E) NOTA

29. What is the units digit of $\sum_{n=0}^{\infty} n!$?

A) 2 B) 4 C) 6 D) 8 E) NOTA

30. Evaluate $1 + \frac{1}{3} + \frac{1}{2} + \frac{1}{9} + \frac{1}{4} + \ldots$

A) $\frac{1}{2}$ B) 1 C) $\frac{3}{2}$ D) 2 E) NOTA