## Discrete Mathematics - Open Solutions

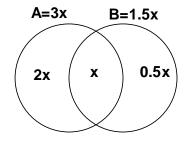
MAO National Convention 2010

The following were changed at the resolution center at the convention: 19 E

1. C  $M^{3} = \begin{vmatrix} 1 & 0 & 0 & 11 \\ 5 & 3 & 9 & 2 \\ 4 & 6 & 11 & 12 \\ 7 & 7 & 15 & 7 \end{vmatrix}$ , and as the number 11 is the element representing the location of (C, C),

there are 11 three-step paths leading from C back to itself.

- 2. B There are 16 vertices  $(N_0)$ , 32 edges  $(N_1)$ , 24 faces  $(N_2)$ , and 8 spaces  $(N_3)$ . 16 - 32 + 24 - 8 = 0.
- 3. B Call  $A \cap B$ , x. Then, A (not including  $A \cap B$ ) must one-third of *A*'s elements to also be in *B*). There are elements in A, so B must have 1.5x elements. including  $A \cap B$ ) must be 0.5x. Adding all 2x + x + 0.5x = 423.5x = 42x = 12So, there are 12 elements in



A d=3

Е

d=2

n d=4 be 2x (in order for a total of 3x Therefore, *B* (not elements, we have:

vertex A

color, B/E

edge must connect

is the "region"

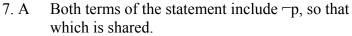
 $A \cap B$ .

x is positive, so the range for y is  $x \le y \le x^2$ . Therefore, the range for z must be 4. A  $0 \le z \le x^2 + x$ . Since the given range is 240, we solve the quadratic  $x^2 + x = 240$ . The only positive solution is 15.

> В d=4

5. B All vertices have an even degree except for (d=3) and vertex C (d=3). Therefore, the new A-C.

6. B The chromatic number is 3. (A/C can be one a second color, and D a third color)

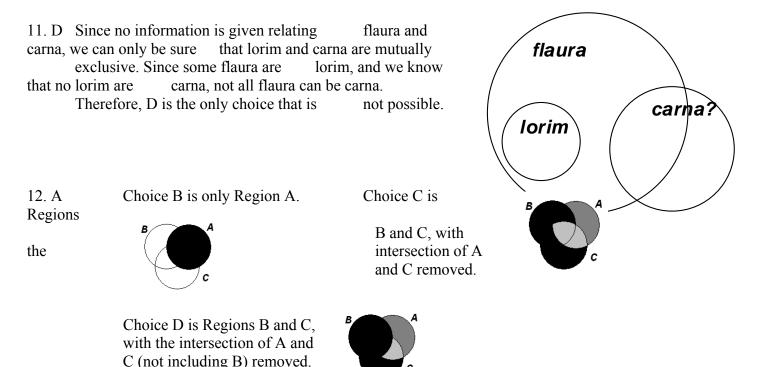


- С d=3 Both terms of the statement include  $\neg p$ , so that р
- 8. C By definition, if  $a \equiv x \pmod{y}$ , then y evenly divides a-x. Likewise, if  $a \equiv y \pmod{x}$ , then x evenly divides *a*-*y*. Therefore, *a* is a number that is divisible by *y* after *x* is subtracted, and vice-versa. As such, *a* must be x+y.

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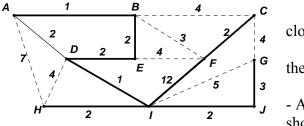
9. D 
$$\frac{\frac{n^2 - n}{2} - \frac{(n-2)^2 - (n-2)}{2}}{\frac{n^2 - n}{2} - \frac{n^2 - 4n + 4 - n + 2}{2}}{\frac{4n - 6}{2} = 2n - 3}$$

10. A The F in the 16<sup>1</sup> digit represents  $15*16^{1}$ , resulting in  $240_{10}$ . The 2 is in the  $16^{0}$  digit, resulting in  $2*16^{0} = 2_{10}$ . Together,  $F2_{16} = 242_{10} = 100(2) + 10(4) + (2)$ . 2+4+2=8.



13. E There are  $2^n$  subsets of a set, including the null set. The subsets can be anywhere from 0 to *n* elements in length. All choices are therefore subsets, so choice E is correct.

14. C The easiest way to find a MSP is by choosing the least expensive edges, not until a MSP is formed (Kruskal's In this case, the MSP is represented by graph. The sum of the edges = 27.
15. A The shortest path is as follows: C - B cost=10. There are a few (seemingly cost 11.



simply closing any circuits, Algorithm). the thick lines in the

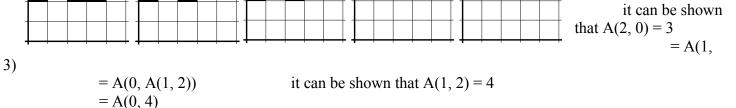
- A - D - I - H, with shorter) paths with

16. B Choice A violates rows 3, 4, and 6 of the table. Choice C violates rows 1 and 4. Choice D violates rows 1, 3, and 4. (most of these result from the fact that if the antecedent is false, the conclusion is irrelevant and the statement must be true)

17. D With logarithmic time-complexity, the increase in speed of the algorithm slows significantly as more complex functions are included. Linear, Exponential, and Quadratic all increase with greater rate at higher algorithm complexity. (To see this, examine the derivative of a standard form of each type of function)

18. C To solve these problems, divide the given integer by increasing powers of 5, adding all the wholenumber parts. So, here, we must divide 2010 by  $5^1$  (402), by  $5^2$  (80), by  $5^3$  (16) and by  $5^4$ (3). 402+80+16+3 = 501.

- 19. A Choice B can be obtained by using the following Rules: II, II, III. Choice C by Rules II, II, III, I, II. Choice D by Rule I.
- 20. B The iterations are shown below. After four iterations, there are two live cells. iteration 0 iteration 1 iteration 2 iteration 3 iteration 4 21. B A(2, 1) = A(1, A(2, 0))

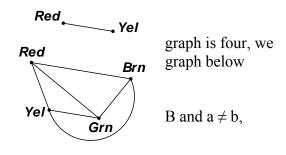


22. B The probability of choosing the correct door on the first try is 1/3 (or, 1/n for n doors). Since the host only leaves one remaining door to switch to, the probability of *that* door being correct is 2/3 (or, (n-1)/n for n doors). It is always beneficial, in the long run, to switch doors.

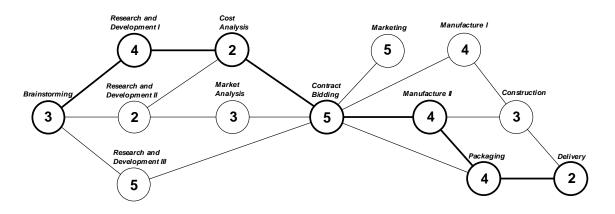
23. C Because the maximum chromatic number for a planar must only need to reach four colors to be satisfied. The accomplishes that.

= 5

24. E None of the operations are commutative. Assuming  $A \neq A$ . A X B  $\neq$  B X A B. a / b  $\neq$  b / a C. a - b  $\neq$  b - a D. a^b  $\neq$  b^a



25. C The shortest path which fits the criteria is as shown. Adding the times of the necessary tasks yields a total of 24 time-units. Any other path would either take longer, or not fit the given criteria.



26. B Reverse Polish Notation calculates based on stacks. Each elements stacks on the others, until an operation is reached, which operates on the two prior elements in the stack. Hence,

8 5 1 3 + \* 2 / - becomes 8 5 4 \* 2 / -8 20 2 / -8 10 --2

27. C This algorithm is different from variations of Djikstra's Algorithm in label is replaced if the "longer" same weight (this is slightly redundant, answer to this question). Once the completed, the graph should look as shown. Vertex D states previous=E and

6 D <sup>prev=E</sup> dist=8 2 Ε Δ prev=C prev=A dist=6 2 dist=2 4 F 3 C prev=B 2 dist=4 1 1 A prev=und. 4 B prev=F dist=3 dist=0 B(Yel) A (Red) C (Red) G (Red) D (Brn)

E (Yel)

F (Brn)

most that, on line 12, the path has the but affects the Algorithm is The label on distance=8.

28. B Create a graph connecting chemicals together. Then, color the graph to determine *can* travel together. Such a graph is shown, and boxes are required.

which *cannot* travel which chemicals shows that three

29. C Rather than actually performing Gauss-Jordan Elimination on each matrix, note that the resultant matrix implies that  $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -4 \\ 1 \\ -2 \end{bmatrix}$ . Multiplying each "left side" by this answer matrix results in each "right  $\begin{bmatrix} -2 & -3 & 4 \\ -3 \end{bmatrix}$ 

side," except for choice C. In fact, choice C should read  $\begin{bmatrix} -2 & -3 & 4 & | & -3 \\ 0 & 8 & 4 & | & 0 \\ 3 & 2 & -5 & | & 0 \end{bmatrix}$ .

30. D The probability of the chip starting in each of the four slots on top is 1/4. For the first and fourth slot, the chip can only fall one direction. For the second and third slots, the probability halves for the chip falling either left or right. For the second set of pegs, the probabilities of the three "slots" are 3/8, 1/4, 3/8. For the third set of pegs, probabilities are 3/16, 5/16, 3/16. For the fourth set of pegs, probabilities are 11/32, 5/16, 11/32. For the fifth set of pegs, probabilities are 11/64, 21/64, 21/64, 11/64. Finally, the probabilities for each receptacle are 43/128, 21/64, 43/128. Hence, the probability for the center receptacle is 21/64, answer D. This Plinko board is similar to the quincunx machine.