For all questions, answer E. NOTA means none of the above answers is correct.

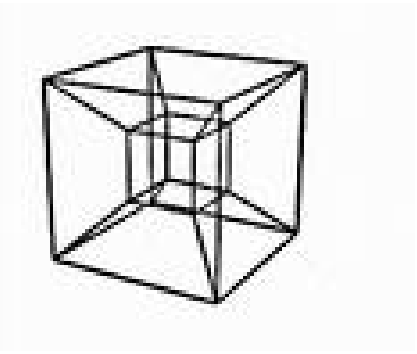
1. The directed multigraph G can be described by adjacency matrix

$$M = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 2 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 2 \\ 1 & 1 & 2 & 0 \end{bmatrix} \end{array}.$$

   Determine the number of three-step paths from node C back to itself.

   A. 1 B. 3 C. 11 D. 15 E. NOTA

2. The Euler Formula of polyhedra states $N_0 - N_1 + N_2 = 2$, where N_k is the number of k-dimensional elements of the polyhedron. This formula is often stated as $V - E + F = 2$. The analogous Euler Formula of polytopes (four-dimensional objects) states $N_0 - N_1 + N_2 - N_3 = c$. Using the given projection of a tesseract (hypercube), determine the correct value for c. (*hint: the number of faces is 24, and don't forget to count the "big" solid for* N_3)



   A. -2 B. 0 C. 1 D. 2 E. NOTA

3. Set A contains twice the number of elements as Set B, and one-third of Set A's elements are also included in Set B. If there are a total of 42 elements in $A \cup B$, then how many elements must be in $A \cap B$?

   A. $\frac{32}{3}$ B. 12 C. 14 D. 24 E. NOTA

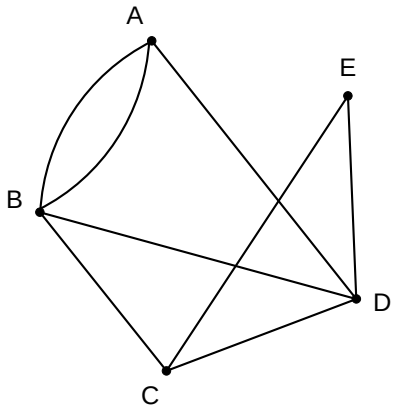
4. The following pseudocode algorithm shows a semi-random number generated from a given seed:

   Input an integer; store it as x
   Randomize an integer between x and x^2; store it as y
   Randomize an integer between y-x and y+x; store it as z
   Display the value of z

   After running the algorithm a large number of times with the same seed, the range of values observed for z is 240. What is the value of the seed?

   A. 15 B. 20 C. 120 D. 240 E. NOTA

5. Consider the graph at right. For the graph to contain an Euler Circuit, an edge would need to be placed between which two vertices?



A. B-D
B. A-C
C. B-E
D. A-E
E. NOTA

6. Determine the chromatic number of the graph above.

A. 2 B. 3 C. 4 D. 5 E. NOTA

7. According the laws of logic, $\neg p \wedge \neg(p \wedge q) \equiv ?$

A. $\neg p$ B. p C. $p \vee q$ D. $\neg q$ E. NOTA

8. If a is the smallest possible positive integer such that $a \equiv x \pmod{y}$ and $a \equiv y \pmod{x}$, then what is the value of a?

A. 1 B. xy C. $x + y$ D. not possible E. NOTA

9. Determine the difference between the number of edges on a complete graph with n vertices and a complete graph with $(n - 2)$ vertices (assume $n \geq 2$).

A. 2 B. $n - 1$ C. $n^2 - n$ D. $2n - 3$ E. NOTA

10. If the hexadecimal number $F2_{16}$ were written in expanded decimal notation $(100a + 10b + c)_{10}$, what would be the value of $a + b + c$?

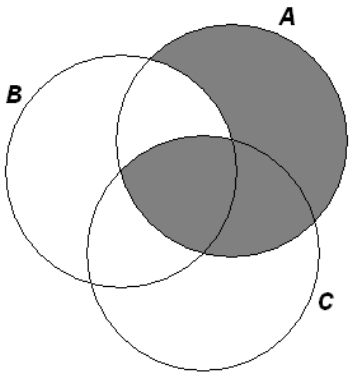
A. 8 B. 11 C. 14 D. 16 E. NOTA

11. Given the following information:
    I. All lorim are flaura
    II. Some flaura are lorim
    III. No lorim are carna

    Which conclusion is not possible?

    A. Some carna are flaura B. All carna are flaura
    C. Some flaura are carna D. All flaura are carna E. NOTA

12. Which of the following statements represents the shaded area of the given Venn Diagram?

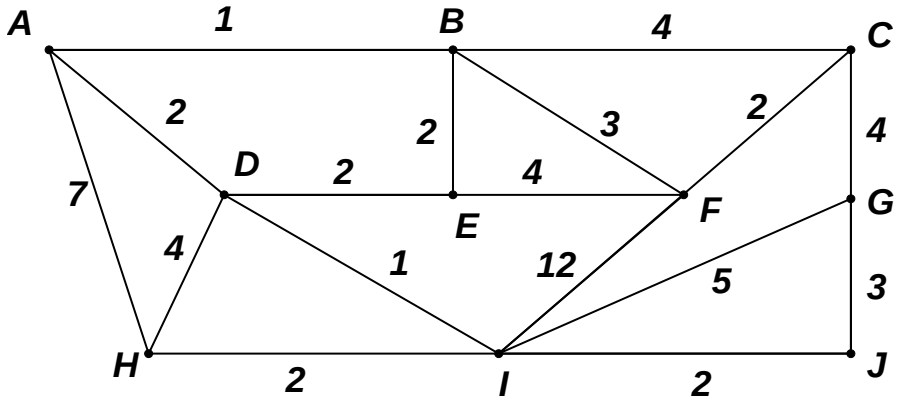


    A. $(A \wedge \sim B \wedge \sim C) \vee (A \wedge C)$
    B. $A \vee (A \wedge C)$
    C. $(B \vee C) \wedge \sim (A \wedge C)$
    D. $(B \vee C) \wedge \sim (A \wedge \sim B \wedge C)$
    E. NOTA

13. Given that $M = \{0, 1, 2, 3\}$, which of the following is not a subset of M?

    A. {0}
    B. {0, 3}
    C. {0, 1, 2, 3}
    D. ø (null)
    E. NOTA

14. Determine the cost of a minimum spanning tree of the following graph, if the tree must include the 12-unit edge.



    A. 17 B. 18 C. 27 D. 60 E. NOTA

15. In the graph above, determine the minimum cost of a path from Vertex *C* to Vertex *H*.

A. 10 B. 11 C. 12 D. 16 E. NOTA

16. Determine which logic statement can be represented by the following truth table:

x	y	z	statement
T	T	T	T
T	T	F	T
T	F	T	F
T	F	F	F
F	T	T	T
F	T	F	T
F	F	T	T
F	F	F	T

A. $(\sim x \wedge y) \rightarrow (y \wedge z)$
B. $(x \wedge \sim y) \rightarrow (y \wedge z)$
C. $(x \wedge z) \rightarrow (\sim x \wedge y)$
D. $(x \wedge z) \wedge y \rightarrow \sim z$
E. NOTA

17. Of the given choices, which class of time-complexity would describe the most efficient algorithm?

A. Linear
B. Exponential
C. Quadratic
D. Logarithmic
E. NOTA

18. Determine how many zeroes are present at the end of the expanded form of 2010!.

A. 201 B. 401 C. 501 D. 916 E. NOTA

19. The MIU System was created by Douglas Hofstadter and published in his book *Gödel, Escher, Bach: An Eternal Golden Braid*. Given a string of Ms, Is, and Us, there are only four rules to symbol manipulation in the formal system
(*x* represents any string of any length):

Rule I: *x***I** -> *x***IU**
Rule II: **M***x* -> **M***xx*
Rule III: *x***III***x* -> *x***U***x*
Rule IV: *x***UU***x* -> *xx*

Given the initial string **MI**, which of the following choices is not a logical derivation, using only the four rules?

A. **MUIIU** B. **MUI** C. **MUIUUIU** D. **MIU** E. NOTA

20. Conway's Game of Life is a cellular automata system, which follows these four rules:

Rule I. Any live cell with fewer than two live neighbors dies
Rule II. Any live cell with more than three live neighbors dies
Rule III. Any live cell with two or three live neighbors continues to live
Rule IV. Any empty cell with three live neighbors becomes live

If the cells located at (1, 3), (1, 4), (2, 4), (3, 3) and (4, 3) are live, how many cells will be live after four iterations of the Game of Life?

A. 0 B. 2 C. 5 D. 14 E. NOTA

21. The Ackermann Function is defined as:

$$A(m,n)=\begin{cases} n+1 & m=0 \\ A(m-1,1) & m\neq 0, n=0 \\ A(m-1,A(m,n-1)) & m\neq 0, n\neq 0 \end{cases}$$

Calculate A(2,1).

A. 4 B. 5 C. 9 D. 125 E. NOTA

22. The following problem, often referred to as the Monte Hall Problem, was published in *Parade* magazine (September 1990):

Suppose you're on a game show, and you're given the choice of three doors: Behind one door is a car; behind the others, goats. You pick a door, say No. 1, and the host, who knows what's behind the doors, opens another door, say No. 3, which has a goat. He then says to you, "Do you want to pick door No. 2?" Is it to your advantage to switch your choice? (Whitaker)

In order to maximize your chance for winning the car, should you stick with your original door (in this case, No. 1), or switch to the remaining door (in this case, No. 2)?

A. stick with original door
B. switch to remaining door
C. odds are even; choice is irrelevant
D. depends on results of previous "contestants"
E. NOTA

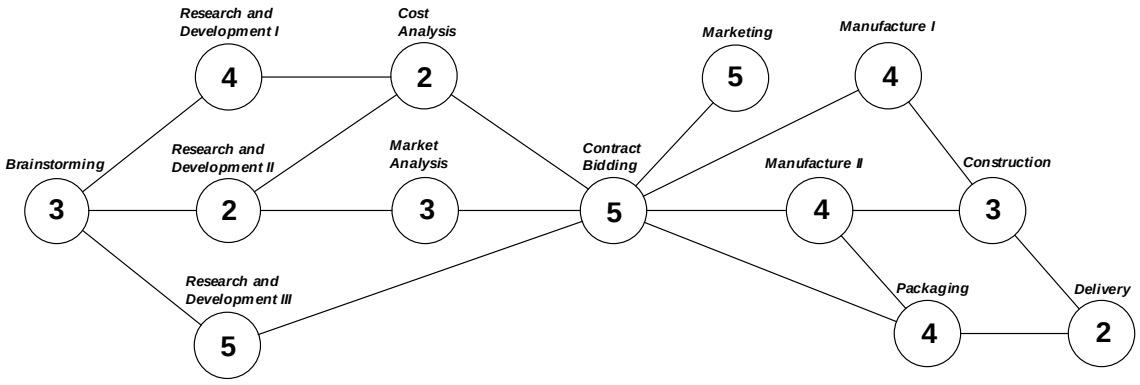
23. A planar graph G contains six nodes and seven edges. What is the maximum possible chromatic number for G?

A. 2 B. 3 C. 4 D. 6 E. NOTA

24. Which of the following operations are commutative?

A. Matrix Multiplication
B. Division
C. Subtraction
D. Exponentiation
E. NOTA

25. Below is PERT (Project Evaluation and Review Technique) chart based on the invention, manufacture, and deployment of a new (fictitious) product. The numbers indicate the number of time-units required to complete each respective task; no task can be started until all preceding tasks have been completed; some tasks can be performed concurrently (so long as one does not precede the other); and time flows from left to right.



What is the shortest number of time-units necessary to complete this chart?

A. 18
B. 19
C. 24
D. 46
E. NOTA

26. Evaluate the following expression, which is written in Reverse Polish Notation (*note: all numbers are single-digit integers)*:

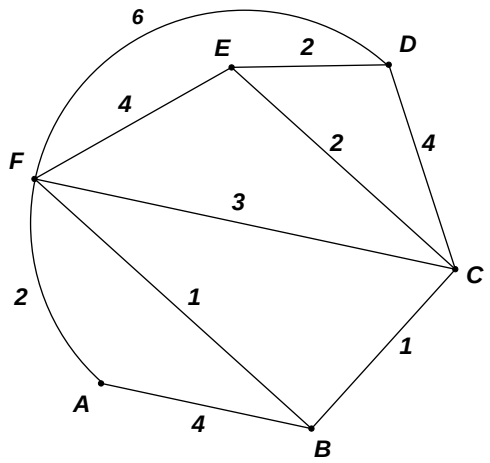
8 5 1 3 + * 2 / -

A. -7/3
B. -2
C. 7/3
D. 23/3
E. NOTA

27. The following is one implementation of Djikstra's Algorithm for finding a MST (minimum spanning tree) of a graph:

```
1   function Dijkstra(Graph, source):
2       for each vertex v in Graph:
3           dist[v] := ∞
4           previous[v] := undefined
5       dist[source] := 0
6       Q := the set of all nodes in Graph
7       while Q is not empty:
8           u := vertex in Q with smallest dist[]
9           remove u from Q
10          for each neighbor v of u:
11              alt := dist[u] + dist_between(u, v)
12              if alt ≤ dist[v]
13                  dist[v] := alt
14                  previous[v] := u
15      return previous[]
```

Using the graph at right, and considering the source to be at vertex *A*, execute Djikstra's Algorithm. At the completion of the Algorithm, which of the following is the correct label for vertex *D*?



A. dist = 8, previous = *F*
B. dist = 8, previous = *C*
C. dist = 8, previous = *E*
D. dist = 7, previous = *E*
E. NOTA

28. Seven chemicals are to be stored in boxes. However, because of the nature of some of these chemicals, they cannot all be stored together. The following is a list of which chemicals can *not* be stored together (not all interactions are mutual):

Chemical A: B, F
Chemical B: C, D, F
Chemical C: B
Chemical D: A, C, G
Chemical E: A, C, D, F, G
Chemical F: no interactions
Chemical G: B, E

What is the minimum number of boxes needed to store these chemicals?

A. 2 B. 3 C. 4 D. 6 E. NOTA

29. Which of the following matrices, when reduced by Gauss-Jordan Elimination, does *not* result in the matrix $\left[\begin{array}{ccc|c} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{array}\right]$?

A. $\left[\begin{array}{ccc|c} 3 & 2 & -1 & -8 \\ -2 & 2 & 5 & 0 \\ 7 & -5 & 2 & -37 \end{array}\right]$

B. $\left[\begin{array}{ccc|c} 2 & 1 & 3 & -13 \\ 1 & 3 & 2 & -5 \\ 3 & 2 & 1 & -12 \end{array}\right]$

C. $\left[\begin{array}{ccc|c} -2 & -3 & 4 & -3 \\ 0 & 8 & 4 & 0 \\ 3 & 2 & -5 & 2 \end{array}\right]$

D. $\left[\begin{array}{ccc|c} 2 & 7 & -3 & 5 \\ 1 & 7 & 1 & 1 \\ -2 & 5 & 6 & 1 \end{array}\right]$

E. NOTA

30. A diagram of a modified "Plinko" board is shown below. Each large dot is a peg in the board. In Plinko, a chip is dropped into the top of the board, and traverses it's way down. At each peg, there is an equal chance of the chip falling left or right.

If a chip is dropped at random at the top of the board shown, what is the probability that it will land in the center receptacle at the bottom of the board? *(hint: find the probability of the chip falling between each pair of pegs)*

A. 1/3
B. 5/16
C. 11/32
D. 21/64
E. NOTA