Probability – Open

## SOLUTIONS

The following were changed at the resolution center at the convention: 14 and 29 thrown out

1. D	$_{7}P_{4} = \frac{7!}{3!} = 840$		
2. D	Probability of getting 5, 6, 7 and 8 heads		
	8 heads = $\frac{1}{2}^{8}$ 7 heads = ${}_{8}C_{1}\frac{1}{2}\cdot\frac{1}{2}^{7} = 8\cdot\frac{1}{2}^{8}$ 6 heads = ${}_{8}C_{2}\frac{1}{2}^{8} = 28\cdot\frac{1}{2}^{8}$ 5 heads =		
	$_{8}C_{4}\frac{1}{2}^{8} = 56$ probability is $\frac{1+8+28+56}{2^{8}} = \frac{193}{256}$		
3. A	The probability of getting at least one 3 is the complement of getting NO 3's.		
	The prob of no three on the die is $\frac{5}{6}$ and the prob of getting no three from the deck is $\frac{48}{52} = \frac{12}{13}$ .		
	Since the events are independent the probability of getting neither is $\frac{5}{6} \cdot \frac{12}{13} = \frac{10}{13}$ the prob of not		
	getting neither is $1 - \frac{10}{13} = \frac{3}{13}$		
4. C	The probability of picking a certain container is $\frac{1}{2}$ . The probability of getting a cherry		
	gumdrop from container 1 is $\frac{1}{2} \cdot \frac{7}{12} = \frac{7}{24}$ . The probability of getting a cherry from the 2 <sup>nd</sup>		
	container is $\frac{1}{2} \cdot \frac{8}{15} = \frac{4}{15}$ . The probability of getting either is $\frac{7}{24} + \frac{4}{15} = \frac{67}{120} \Rightarrow a+b=187$		
5. A	The total area is Region 1 + Region 2		
	Region 2 = one fourth of a circle with radius $\sqrt{2} = \frac{1}{4}\pi \left(\sqrt{2}\right)^2 = \frac{\pi}{2}$		
	The outside area is Region 1: $4 - \frac{\pi}{2}$ . The probability is the ratio.		
	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		
	4 8		
6. C	The unseen cards do not effect the probability		
7 C	The zero can be in the $2^{nd}$ column or the $3^{rd}$ . There are $9*1*9$ for the $2^{nd}$ column and $9*9*1$ for		
	the $3^{rd}$ column. $81+81 = 162$		
8 A	$P(B   A) = \frac{P(A \cap B)}{P(A)}  P(A \cap B) = P(B   A)P(A) = \frac{7}{10 * 4} = \frac{4}{10} = \frac{2}{5}$		
9. C	There are 3!=6 ways to rearrange 6-2-1, 3! ways to rearrange 5-3-1, 3 ways to rearrange 5-2-2, 3		
	ways to rearrange 4-4-1, 6 ways to rearrange 4-3-2 and one way to rearrange 3-3-3 total $6 + 6 + 2 + 2 + 6 + 1 = 25$ there are $602$ ways to reall diag, area $25/216$		
10 F	total $0+0+5+5+0+1 = 25$ there are 0°5 ways to foll dice prob = $25/210$ The sum of the any row of Deceal's triangle must be of the form $2^n$ and so must be divisible by		
10. L	4. The divisibility rule for 4 is that the last two digits must be divisible by 4. None of the		
	solutions have that property.		

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MAO National Convention 2010

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11 D				
11. D	ROYGBIV alphabetized is <b>BGIORVY</b> . There are $7^{6}5^{4}$ total rearrangements. There are $6^{5}4^{4} - 120$ for each of these letters. So the answer must start with <b>G</b> = 211-120 - 91			
	$0^{-}5^{-}4 = 120$ for each of these letters. So the answer must start with <b>G</b> : 211-120 = 91 G can be used with <b>PIODVV</b> . Each of these letters has $5*4-20$			
	G can be used with <b>BIORVY</b> . Each of these letters has $5^{+}4=20$ .			
	GB has 20, GI has 40, GO has 60, GR has 80 and GV contains 91. So we need to use GV**			
	and have 11 more to go. $GV^{**}$ can be used with BIORY, each of which has 4			
12.4	GVB* leaves 7, $GVI$ leaves 3 $GVO + 3 = GVOR$			
12 A	$\frac{4! = 120}{2}$			
13 B	This is asking for the number of combinations			
14 B	There are 19 perfect squares in the first 400 numbers. $2^2$ , $3^2$ ,, $20^2 = 400$ . So we must			
	remove 19 numbers from the list. There are 7 perfect cubes $2^3=8,,7^3=343$ but 64 is both a			
	perfect cube AND a perfect square so we remove 6 more numbers from the list. There are 19+6			
	= 25 missing numbers <= 400. Since the list starts at 2, Total-25 = 400: 425			
15. D	Since there are 2 L's 2 S's and 3 E' $\rightarrow \frac{10!}{-10!} = \frac{10!}{-10!}$			
	Since there are 2 L s, 2 s s and 5 E $-\frac{1}{2!2!3!} - \frac{1}{4!}$			
16 C	We get the expected value by weighing each value by its probability and summing			
	$(1)^3$ 1 1 3			
	Your chance of getting 3 matches is $\left \frac{1}{6}\right  = \frac{1}{216}$ expected payoff is $3 \cdot \frac{1}{216} = \frac{3}{216}$			
	(6) 216 216 216			
	Matching 2: $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{5}{2} = \frac{5}{2}$ this can happen 11x 1x1 x11 so $\frac{15}{2} \cdot 2 = \frac{30}{2}$			
	6 6 6 - 216, this can happen 11x,1x1,x11 so $216 - 216$			
	Matching 1, 1, 5, 5, 25 this can be mand three many 1 m m1 m m1 m m1 m 75, 75			
	Matching 1: $-\cdot - \cdot = \frac{1}{216}$ this can happened three ways 1xx, x1x, xx1 so $\frac{1}{216} \cdot 1 = \frac{1}{216}$			
	N + 1 = 0 5 5 5 125 LOSE #1 125			
	Matching 0: $-\cdot = -$ you LOSE \$1 so $-\frac{16}{216}$			
	3 30 75 125			
	Expected value is the sum: $\frac{3}{216} + \frac{30}{216} + \frac{75}{216} - \frac{125}{216} = -0.7870 \approx08$			
15.1	So for a dollar bet you would expect to receive \$0.92			
17 A	A player is most likely to be attracted to a game like this, if he thinks he can win. Excitement is			
	generated if he sees a lot of winning. The VARIABILITY of the results of this game causes a			
	lot of money to change hands. The variability helps disguise the fact that the player tends to			
	lose. The operator needs to make money. The operator wants an <b>Expected Value</b> in his favor			
	so that he can be sure that if a lot of games are played, the money will most like move in his			
	direction. The operator expects to average 8 cents per game.			
18 A	14 + 20 + 13 - 8 - 10 - 5 + x = Class			
	24 + x = Class			
	$x/(24 + x) = 1/7 \rightarrow 7x = 24 + x \rightarrow 6x = 24 \rightarrow x = 4 \rightarrow Class Size = 28$			
	Sum of digits is 10			
19 D	if the cardinality of a finite set S is <i>n</i> , then the cardinality of the Power Set of S is $2^n \ 2^5 = 32$			
20 D	Infinite Geometric Series: Prob of plays in order Joe: 1/6 Bob 5/6*1/6 Saul 5/6*5/6*1/6			
	Joe 5/6*5/6*1/6, Bob 5/6*5/6*5/6*5/6*1/6, Saul 5/6*5/6*5/6*5/6*5/6*1/6			
	Saul's pattern is $5/6*5/6*1/6$ , $(5/6)^3 * (5/6*5/6*1/6)$ so $a = (5/6*5/6*1/6)$ and $r = (5/6)^3$			
	$sum = a/(1-r) \rightarrow (5/6*5/6*1/6)/(1-(5/6(^3) = 25/91))$			
21 C	Consider a set of 15 1's and $4 +$ 's. There are $19 = 15+4$ symbols and we want to find all the			
	ways to place the +'s. The problem reduces to 19 choose 4.			

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22 B	$ \frac{n!}{nC_4 = 5\binom{n}{n}} \frac{n!}{4!(n-4)!} = \frac{5n!}{5!(n-5)!} \frac{n!}{\cancel{4!}(n-4)!} = \frac{\cancel{2}n!}{\cancel{4!}(n-5)!} \frac{1}{(n-4)(n-5)!} = \frac{1}{(n-5)!} \frac{1}{(n-5)!} \frac{1}{(n-5)!} = \frac{1}{(n-5)!} \frac{1}{(n-5)!} \frac{1}{(n-5)!} \frac{1}{(n-5)!} = \frac{1}{(n-5)!} \frac{1}{($			
	n-4 = 1 $n=5$			
23 B	If the odds of winning are 3 to 8 t	he probability is $3/(8+3) = 3/11$ . The other players		
20 0	n the olds of winning are 5 to 8, the probability is $5/(6+5) = 5/11$ . The other players			
24 C	This is the femous Monty Hell problem. As herd as it is to believe, the proper strategy is to			
24 C	switch. You change of winning increases to $2/3$			
25 \	switch. You chance of winning increases to 2/3			
23. A	$-\frac{1}{2}, -\frac{4}{2}, -\frac{7}{2}, -\frac{10}{2}$ 10 4.7 7 7 7			
	$\int \frac{3}{3} \frac{3}{3} \frac{3}{3} (8y)^{-\frac{10}{3}} (-3x)^3 \text{ coefficient} \frac{4 \cdot 7}{3} 2^{-10} (3)^3 \frac{4 \cdot 7}{3} = \frac{7}{3} = \frac{7}{3}$			
	$1 \cdot 2 \cdot 3 \qquad (3) $			
26. B	$_{8}C_{3} \cdot _{8}C_{5} \frac{8!}{3!5!} \cdot \frac{8!}{5!3!} = \left(\frac{8 \cdot 7 \cdot 6}{1 \cdot 2 \cdot 3}\right)^{2} = 56^{2} = 3136$			
27. C	A = 5! B = 4! C=4!/2 A/B + B/C = 5!/4! + 4!/(4!/2) = 5 + 2 = 7			
28 B	Solve this one geometrically.			
	Your arrival times can be			
	modeled by	The shaded region represent the times when you might		
		meet. The area of the total box is $60^2$ , the area of the		
		$15^2$		
		triangles are $\frac{45}{2}$ . So the probability of falling		
		2		
		into the shaded area is $\frac{60^2 - 45^2}{1000000000000000000000000000000000000$		
		$60^2$ $15^2$ $4^2$ $16$		
	0 15 30 45 60			
29 E	No event can be mutually exclusiv	e AND independent.		
30 A				
		UC .		
	7			
	.82			
	Guan 08 .	.7 🕒		
	succua_			
	.3			
	.96 .1			
	03 pt 1			
	TRANS			
	PLANT			
	,01 .48 <b>~</b>			
	There are 3 paths to survival .96*.82 .96 * .08 * .7 and .03 * .52			
	The probability of survival is the sum if these terms			