#### **MAO** National Convention 2010

### **Probability – Open**

For all questions, answer E. "NOTA" means none of the above answers is correct. **Calculator use – NO calculators will be permitted on any test other than the Statistics topic test.** The word "deck" refers to a standard 52 card deck that is well shuffled.

1	Evaluate $_7P_4$			
	A. 24	B. 3	5	C. 120
	D. 840	E. N	JOTA	
2	A fair coin is flipped 8 times. A. $\frac{1}{2}$ D. $\frac{93}{256}$	B1	probability of getting more hear 53 100 NOTA	ds than tails? C. $\frac{23}{64}$

3 A card is drawn from a standard deck and a die is tossed. What is the probability of obtaining at least one 3?

A.	3	В.	4	C.	10
	13		13		13
D.	12	E.	NOTA		
	39				

4 Two identical bowls contain gumdrops. The first box contains 7 cherry and 5 orange gumdrops. The second bowl contains 8 cherry gumdrops and 7 grape gumdrops. The probability that a gumdrop

chosen at random from a randomly selected bowl is cherry is  $\frac{a}{b}$ , where a>0, b>0 and GCD(a,b)= 1.

Find $a+b$ .			
A. 14	B. 63	C.	187
D. 1122	E. NOTA		

5 If two numbers between 0 and 2 (inclusive) are chosen at random what is the probability that the sum of their squares exceeds 2?

А.	$8-\pi$	В.	1	C.	$\pi$
	8		$\overline{8}$		$\overline{2}$
D.	1	E.	NOTA		
	$\overline{2}$				

6 In two-person Gin Rummy, the dealer deals each player ten cards, face down, one at a time. After the 20 cards are dealt what is the probability that the next card will be a King?

E.	21 NOTA	$\overline{13}$
E.	NOTA	
exactly	one zero?	
В.	100	C. 162
E.	NOTA	
	B.	exactly one zero? B. 100 E. NOTA

7

8	If $P(A) = \frac{7}{10}$ and $P(B A) = \frac{4}{7}$ , Fin	I the $P(A \cap B)$	
	A. 2	B. 40	C. 9
	5	49	$\overline{70}$
	D. 61	E. NOTA	
	70		
9	What is the probability of getting a	otal of 9 when three dice are three	own?
	A. 1	B. 5	C. 25
	$\frac{1}{9}$	36	216
	D. 13	E. NOTA	
	108		

10	Let	N be the sum	of the elements in the 3	0 row of Pascal's Triangle.	What is	N (mod 100)?
	А.	21	B.	22	C.	25
	D.	26	E.	NOTA		

#### The following description should be used for problems 11-13.

Roy has a large number of colored blocks. The blocks come in the following colors: Red, Orange, Yellow, Green, Blue, Indigo and Violet. Roy creates every possible 4-block rearrangement of distinct colors, sorts them by alphabetizing them by color name and then spreads them out on a long table.

11 The first rearrangement is BGIO. Roy's favorite number is 211. Which rearrangement is in position 211?

А.	GIBY	В.	GOBR	C.	GRIV
D	CUOD	T	NOTA		

- D. GVOR E. NOTA
- 12 Roy's favorite colors are Green, Blue, Indigo and Violet. How many of the rearrangements contain all four of those colors?
  - A. 24 B. 35 C. 120
  - D. 840 E. NOTA

Roy decided to separate all the rearrangements into sets with the following property: Each set would contain only rearrangements with the same set of four colors. How many different sets would he have?
A. 24
B. 35
C. 256
D. 840
E. NOTA

- 14 The list 2,3,5,6,7,9,10,... contains every positive integer that is not a perfect square or a perfect cube. What is the 400<sup>th</sup> element of this list.
  - A. 424 B. 425 C. 426
  - D. 427 E. NOTA

3!4!

- 15 How many distinguishable arrangements are there for the letters in "BESTSELLER"? A. 10! B. 10! C. 10!
  - D. <u>10!</u> E. NOTA

4!

16 Chuck-a-Luck is a game often played at carnivals. A player bets \$1 on any one of the numbers 1,2,3,4,5 or 6. Three dice are rolled. If the player's number appears on one, two or three dice, he receives \$1, \$2 or \$3 respectively, plus his \$1 bet back, otherwise he loses his dollar. To the nearest penny, what is his expected return on each play?

Α.	\$1.08	В.	\$0.99	C.	\$0.92
D.	\$0.89	E.	NOTA		

17 Chuck-a-Luck is a very popular game. For a game to be successful there must be elements of the game that appeal to either the player and/or the person operating the game. Select the pair of terms that best describes the attraction of the Player and then the Operator.

A.	Variability-Expected Value	В.	Expected Value-Variability	C.	Skew-Expected Value
D.	Mean-Skew	E.	NOTA		

18 In a class, 14 people like Avocados, 20 people like Bananas, 13 people like Cucumbers. 8 people like Avocados and Bananas, 10 people like Bananas and Cucumbers while only 5 people like Avocados and

Cucumbers. The probability that a person likes Apples, Bananas and Cucumbers is  $\frac{1}{7}$ . If *n* is the

number of people in the class, what is the sum of the digits of *n*?

A. 10	B.	8	C.	6
D. 4	E.	NOTA		

 19
 If set  $S = \{1, 2, 4, 8, 16\}$  how many subsets does S have?

 A. 5
 B. 10
 C. 120

 D. 32
 E. NOTA

20 Joe, Bob and Saul are playing a game where they roll two dice until someone gets a 7. If the players roll in the order above and take turns, what is the probability that Saul wins?

A.	1	В.	10	C.	71
	3		33		216
D.	25	E.	NOTA		
	<u>91</u>				

21 In how many ways can 15 be written as the sum of five non-negative integers? (Assuming order matters)

A.  ${}_{15}C_5$ D.  ${}_{19}C_5$ B.  ${}_{15}C_4$ C.  ${}_{19}C_4$ E. NOTA

C. 7056

22	Solve for <i>n</i> such that ${}_{n}C_{4} = 5({}_{n}C_{5})$		
	A. 3	B. 5	C. 6
	D. 10	E. NOTA	

23 Joyce and Jill are playing tennis. If the odds of Joyce winning are 3 to 8, what is the probability that Jill will win?

A.	3	В.	8	C.	3
	$\overline{8}$		11		11
D.	5	E.	NOTA		
	$\overline{8}$				

24 Suppose you're on a game show and you're given the choice of three doors. Behind one door is a car; behind the others, goats. The car and the goats were placed randomly behind the doors before the show. The rules of the game show are as follows: After you have chosen a door, the door remains closed for the time being. The game show host, who knows what is behind the doors, now has to open one of the two remaining doors, and the door he opens must have a goat behind it. If both remaining doors have goats behind them, he chooses one randomly. After the host opens a door with a goat, he will ask you to decide whether you want to stay with your first choice or to switch to the last remaining door. You have two choices: 1. Stay with the door you originally chose. 2. Switch to the other door. What are the probabilities for Choice 1 and Choice 2?

A.	Stay = $\frac{1}{3}$ Switch= $\frac{1}{3}$	B.	Stay = $\frac{1}{2}$ Switch= $\frac{1}{2}$	C.	Stay = $\frac{1}{3}$ Switch = $\frac{2}{3}$
D.	Can not be determined	E.	NOTA		

#### 25

Wh	at is the f	Fourth coefficient of the expansion $(8y - 3x)^{-\frac{1}{3}}$		
А.	7	B. 14	C.	5
	1536	$-\frac{1}{81}$		$-\frac{1}{81}$
D.	7	E. NOTA		
	41472			

26 How many committees consisting of 3 Democrats and 5 Republicans can be chosen from a group of candidates which includes 8 Democrats and 8 Republicans?

A.	112	В.	3136
D.	12870	E.	NOTA

I have 5 unique keys. If A = the number of ways I can arrange them in a line, B=the number of ways I 27 can arrange them in a circle and C= the number of distinguishable ways I can arrange them on a key ring, what is  $\frac{A}{B} + \frac{B}{C}$ ? C. 7 A. 1 Β. 11 2 E. NOTA

4 of 6

D. 258

28 You and a friend agree to meet at your favorite fast-food restaurant between 5:00 and 6:00 P.M. Assume that each of your arrival times is random within that hour. The one who arrives first will wait 15 minutes for the other, after which the first person will leave. What is the probability that the two of you will actually meet?

	A. 1	В.	7	C.	9
	$\overline{4}$		$\overline{16}$		$\overline{16}$
	D. 1	E.	NOTA		
	$\overline{2}$				
	-				
29	Consider the following events:				
	E1 = students on the swimming team	ı			
	E2 = students on the debate team				
	E3 = students on the water polo team	ı			
	E4 = rolling a die				
	E5 = drawing a card from a standard	52 0	card deck		
	E6 = spinning a spinner with 6 choic	es {	1,2,3,4,5,6}		
	Which of the following pairs of even	nts is	both Mutually Exclusive AND	Inde	ependent
	A. E1 and E2	В.	E1 and E3	C.	E4 and E5
	D. E4 and E6	E.	NOTA		

- 30 After a traumatic blow, Alex's kidneys have failed so he can not survive unaided. Only about 52% of patients survive for 3 years with kidney dialysis. Transplant operations usually succeed. After 1 month, 96% of the transplanted kidneys are functioning. Three percent fail to function, and the patient must return to dialysis. The remaining 1% of patients die within a month. Patients who return to dialysis have the same chance (52%) of surviving 3 years as if they hadn't attempted a transplant. Of the successful transplants, however, only 82% continue to function for 3 years. Another 8% of these patients must return to dialysis, and 70% of these survive to the 3-year mark. The remaining 10% of "successful" patients die without returning to dialysis. Which of the following best represent Alex's 3-year survival rate?
- A (.96)(.82)+(.96)(.08)(.70)+(.03)(.52) B, (.96)(.82)+(.08)(.70)+(.03)(.52)
- C. (.96)(.82) + (.96)(.08)(.70) + (.03)(.48) D. (.52)(.82) + (.96)(.08)(.70) + (.03)(.52)
- E. NOTA

#### Answers

1113 WCI 5					
1.	D	840	16.	С	
2.	D		17.	А	
3.	А		18.	А	
4.	С		19.	D	
5.	А		20.	D	
6.	С		21.	С	
7.	C		22.	В	
8.	А		23.	В	
9.	С		24.	С	
10.	Е	must be divisible by 4	25.	А	
11.	D		26.	В	
12.	А		27.	С	
13.	В		28.	В	
14.	В	425	29.	Е	
15.	Е	$\frac{10!}{2!2! \neq 4!}$	30.	А	
		3!2!2!			

А	8
В	6
С	8
D	5
Е	3
	30