1. $LA=\pi rs=\pi(3)(5)=15\pi$; $V=\pi r^2h=\pi\left(\frac{1}{\sqrt{\pi}}\right)^2(15\pi)=15\pi$. **A**

2. Figure A is 2 equal cones where $r=\frac{1}{2}s\sqrt{3}$ and $h=\frac{1}{2}s$. The volume of one of these cones is $V_{cone}=\frac{\pi}{3}r^2h=\frac{\pi}{3}\left(\frac{1}{2}s\sqrt{3}\right)^2\left(\frac{1}{2}s\right)=\left(\frac{\pi}{24}\right)\left(3s^3\right)$, so $V_A=2\left(\frac{\pi}{24}\right)\left(3s^3\right)=\left(\frac{\pi}{24}\right)\left(6s^3\right)$. Figure B is a cone where $V=\frac{\pi}{3}r^2h=\frac{\pi}{3}\left(\frac{1}{2}s\right)^2\left(\frac{1}{2}s\sqrt{3}\right)=\frac{\pi}{24}\times s^3\sqrt{3}$. Ratio = $\frac{\frac{\pi}{24}\times 6s^3}{\frac{\pi}{24}\times s^3\sqrt{3}}=2\sqrt{3}$. **D**

3. An octahedron has 8 faces, 6 vertices. The figure with 6 faces, 8 vertices is a hexahedron (cube). **B**

4. $SA=4\pi r^2=4\pi(100)=400\pi$. **B**

5. Side length of the pyramid is *r*, altitude of one face is *l*, side length of base is *x*. Then by right triangles within the pyramid, $\left(\frac{1}{2}x\right)^2+(3)^2=l^2$ and $l^2+\left(\frac{1}{2}x\right)^2=r^2$. Simplify: $\frac{1}{2}x^2+9=r^2$, so $x^2=2r^2-18$. The volume of the figure is $V=\frac{1}{3}Bh=\frac{1}{3}(x^2)(3)=x^2=2r^2-18$. **C**

6. $V=xyz$, $V_0=\left(\frac{4}{3}x\right)\left(\frac{1}{4}y\right)(az)=\frac{a}{3}xyz$. Since $V_0=V$, $\frac{a}{3}xyz=xyz$, and $a=3$. This is 300% of the original, or a 200% increase from the original. **E**

7. *x* is always 1, *y* is always 2, making their intersection a point in space. Extended to three dimensions where *z* is anything makes a line parallel to the z-axis. **B**

8. The question asks for the volume of the figure, which is the difference between the volume of the cylinder with radius 6 and height 2 and the volume of the cylinder with radius 5 and height 2. $V=2\pi(36-25)=22\pi$. **D**

9. This question is not as hard as it looks. The ellipse that is formed by this plane is simply a circle. Its foci thus are joined at its center. The radius is 3 (passes through the point (4,2,0) and centered at (1,2,0)) and the height is 9. The ratio of the radius of the circle in question to the radius of the base is shown by the proportion $\frac{(9-5)}{r}=\frac{9}{3}$. Thus, $r=\frac{4}{3}$ and the area is $\frac{16\pi}{9}$. **B**

10. Combination of a cylinder and a cone. The cone's height is $5\sqrt{3}$, so $V_{cone}=\frac{\pi}{3}(5)^2(5\sqrt{3})=\frac{125\pi\sqrt{3}}{3}$. The height of the cylinder is $11-5\sqrt{3}$, so $V_{cylinder}=\pi(3)^2(11-5\sqrt{3})=99\pi-45\pi\sqrt{3}$.

$V_{cylinder}+V_{cone}=99\pi-45\pi\sqrt{3}+\frac{125\pi\sqrt{3}}{3}=99\pi-\frac{10\pi\sqrt{3}}{3}$. **D**

11. This is just knowing formulae and being able to simplify: $\pi r(s+r)=2\pi r(r+h_{cyl})$; $s^2=r^2+h_{cone}{}^2=r^2+(3h_{cyl})^2$; $\pi r\left(\sqrt{r^2+9h_{cyl}{}^2}+r\right)=2\pi r(r+h_{cyl})$; $\sqrt{r^2+9h_{cyl}{}^2}+r=2r+2h_{cyl}$;

$\sqrt{r^2+9h_{cyl}{}^2}=r+2h_{cyl}$; $r^2+9h_{cyl}{}^2=r^2+4rh_{cyl}+4h_{cyl}{}^2$; $5h_{cyl}{}^2=4rh_{cyl}$; $5h_{cyl}=4r$;
$\frac{5}{4}h_{cyl}=\frac{5}{12}h_{cone}=r_{cone}$; $h_{cone}=\frac{12}{5}r_{cone}$. **B**

12. Diagonal of cube = $\sqrt{3s^2}=6\sqrt{6}$. We want s^2; square both sides: $3s^2=216$ and we get 72. **B**

13. The resulting figure is an octahedron whose side length is $\frac{\sqrt{2}}{2}$ times that of the side length of the cube and whose height is half the side length of the cube (using 45-45-90 triangles within the cube), so $s=4\sqrt{2}$ and $h=4$. An octahedron is two opposed pyramids with square bases, so its volume is $\frac{2}{3}Bh$. This results in a total volume of $\frac{2}{3}\left(4\sqrt{2}\right)^2(4)=\frac{256}{3}$. Area available for packing is this value subtracted from the total volume of the cube (512), which is $\frac{1280}{3}$. **C**

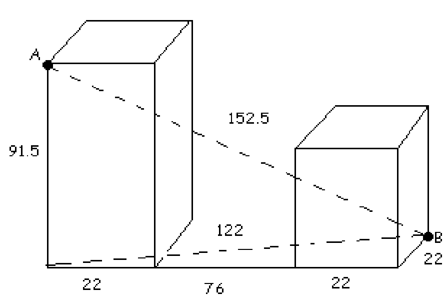
14. The resulting figure is a cylinder with radius 4 and height 2 (be careful-- it is not revolved around the *y*-axis!); Surface Area = $2\pi r(r+h)=2\pi(4)(4+2)=48\pi$. **A**

15. Another question mainly dealing with formulae and simplification: $2b=8a$, so $b=4a$. $3a+2b=ra$. $3a+2(4a)=11a=ra$, so $r=11$. The circumference of a great circle would be 22π. **D**

16. $xy=\sqrt{35}$, $yz=7\sqrt{5}$, xz is unknown, and $xyz=35$. $x^2y^2z^2=(\sqrt{35})(7\sqrt{5})(?)=(35)(35)$. Simplifying gives a result of $\frac{35}{\sqrt{7}}=5\sqrt{7}$ for the unknown, the area of one face. The sum of the two previously unknown faces is $10\sqrt{7}$. **E**

17. The volume of the cone is $V=\frac{\pi}{3}r^2h=\frac{\pi}{3}(4)(8)=\frac{32\pi}{3}$. The volume of each scoop of ice cream is $V=\frac{4\pi}{3}r^3=\frac{4\pi}{3}(1)^3=\frac{4\pi}{3}$. It will take eight scoops to fill the volume of the cone. **C**

18.



The horizontal distance between the ends of the towers is 22+22+76=120. Factoring out 120 gives 60x2, and likewise with 22 gives 11x2. The Pythagorean triple is 11-60-61, so the diagonal is 61x2=122. Factoring 91.5 gives 61x1.5, while for 122 we know it is 61x2. The triangle here is 1.5-2-2.5, the same ratio as a 3-4-5 triangle. Thus, the final distance is 61x2.5=152.5. **C**

19. We are told $Bh_{prism}+\frac{1}{3}Bh_{pyr}=147$ and $h_{pyr}=4h_{prism}$. Substituting to get consistent variables we get $\frac{7}{3}Bh_{prism}=147$. $Bh_{prism}=63$, and back substituting pyramid for prism and multiplying by one-third (in order to get the volume of the pyramid), we get $\frac{1}{3}Bh_{pyr}=\frac{63\times4}{3}=84$. **C**

20. The amount of cubes making up the large cube is 8^3, or 512. The unpainted cubes are a 6x6x6 block on the inside = 216 cubes. The cubes that are "blue-touched" are the 64 on each face = 128. Subtracting both of these numbers from 512 gives us the number of cubes touched by red, green, or red and green: 168. (Double-check this figure by adding the one-color cubes (6x6x4) and the red-green cubes (1x6x4).) 168:216 reduces to 7:9. **B**

21. The volume of any prism is the area of its base multiplied by its height. Once the coordinates are in order by vertices, the area can be found by the following method:

0 0
-1 5
-1 7
6 7 → A = ([0(-1)+5(-1)+7(6)+7(5)+2(0)] – [0(5)+(-1)(7)+(-1)(7)+6(2)+5(0)]) /2
5 2 = [72-(-2)]/2=74/2. *A* = 37.
0 0

Notice that the other coordinates are all 9 units above each vertex, so the height is 9. So, the volume of the figure is (37)(9) = 333. **D**

22. The graph is a circle. Rotating 180 degrees makes a sphere (NOT a hemisphere!) **A**

23. One sheet of paper is 1/100 cm. The book contains 500 sheets, so it is 5 cm. thick. **C**

24. The full cylinder should have volume 90π, so we know the depression takes up 10π cubic cm. of space. The volume of the sphere is 45π, so the ratio is $\frac{10\pi}{45\pi} = \frac{2}{9}$. **E**

25. $\frac{|3(1)-4(4)-1(1)-12|}{\sqrt{3^2+(-4)^2+(-1)^2}} = \frac{|-26|}{\sqrt{26}} = \sqrt{26}$. **B**

26. A polyhedron that has two or more different polygons as faces is known as an Archimedean solid **(A)**. A Platonic solid describes only the five convex regular polyhedra; a polyhedral compound is a combination of two polyhedra (they have "spikes"); a Johnson solid is a convex polyhedron which by its analytic definition excludes Platonic solids and Archimedean solids.

27. Considering that the cylinder could be any height and any radius, as well as have its bases in any plane, none of the points listed necessarily have to be within or on the cylinder. **A**

28. $V = \pi r^2 h$; $800\pi = \pi(4)^2 h$; $h = 50$. **C**

29. $10\pi = 2\pi r(r+h) = 2\pi(1)(1+h)$, $h = 4$. The length of the base of the prism is $\sqrt{2}$, so the surface area is $4[(4)(\sqrt{2})] + 2[(\sqrt{2})(\sqrt{2})] = 4 + 16\sqrt{2}$. **D**

30. When t = 5, the vertex angle is 120 degrees. This makes a 30-60-90 triangle within the cone, giving a radius of $\frac{5}{2}\sqrt{3}$. The area of this cone's base is $\frac{75\pi}{4}$. When t = 3, the vertex angle is 90, forming a 45-45-90 triangle, giving a radius of $\frac{5}{\sqrt{2}}$. The area of this cone's base is $\frac{25\pi}{2}$. The difference is $\frac{25\pi}{4}$. **B**