- 1. **B** Let x = the original price. Then $x 0.10x = 5.85 \rightarrow 0.90x = 5.85 \rightarrow x = 6.50$
- 2. **C** If x is the first value of the Richter Scale and y is the second value, then we can determine how many times stronger the second is by using $\frac{10^x}{10^y}$. So $\frac{10^{5.9}}{10^{5.6}} = 10^{0.3} = 10^{\frac{3}{10}} = 10^{\frac{3}{10}}$
- 3. C The following proportion (comparing tagged fish to total population) can be used to solve the problem: $\frac{120}{x} = \frac{80}{550} \rightarrow x = \frac{(550)(120)}{80} = 825$
- 4. A This problem can be drawn as a right triangle with legs 6 and 10 and ladder length (hypotenuse) of x. So $x^2 = 10^2 + 6^2 = 136 \rightarrow x = \sqrt{136} = 2\sqrt{34}$
- 5. **D** The distance from the center of the ellipse to one focus is $c^2 = 10000 3600 = 6400 \rightarrow c = 80$. So the distance between the two foci is 2(80) = 160.
- 6. **A** $f = kd \rightarrow 25 = k(-10) \rightarrow \frac{-5}{2} = k$. So the equation is $f = \frac{-5}{2}d$. So $f = \frac{-5}{2}(-38) = 95$
- 7. C First, determine the total number of hours of the trip: $\frac{1000}{x} = 90 \rightarrow x = \frac{100}{9}$. So the number of hours

on the return trip was $\frac{100}{9} - 10 = \frac{10}{9}$. So the average speed on the return trip was $\frac{500}{10/9} = 450$

8. **B** An equation relating hat size (x) to head size (y) is $y - 20 = \frac{20 - 19\frac{1}{2}}{6\frac{1}{2} - 6\frac{3}{8}}(x - 6\frac{1}{2}) \Rightarrow y = 4x - 6$. So for

a hat size of 9, the head size is y = 4(9) - 6 = 30

- 9. **B** From the previous problem y = 4x 6. So A+B = 4 + (-6) = -2
- 10. A Whitney's quiz average is $\frac{75+85}{2} = 80$. Let x be the grade she needs on her final:

 $(0.4)(80) + 0.6x = 50 \rightarrow 0.6x = 18 \rightarrow x = 30$

- 11. **B** $\log(6.3 \times 10^{-5}) = \log(6.3) + \log(10^{-5})$. We can determine that $0 < \log(6.3) < 1$ and that $\log(10^{-5}) = -5$. So $-5 < \log(6.3) + \log(10^{-5}) < -4$. So $4 < -(\log(6.3 \times 10^{-5})) < 5$. So the interval is [4,5].
- 12. **B** There was originally .9(100) = 90g of pure gold. Now the percentage of gold in the alloy is $\frac{90}{100} = 60\%$. So the karat rating is 0.6(24) = 14.4 which rounds to 14.
 - $\frac{90}{150} = 60\%$. So the karat rating is 0.6(24)=14.4 which rounds to 14.
- 13. **D** Team E has 4 wins out of <u>6 games</u> (they cannot play themselves so there are only 6 games). So the win percentage is $\frac{4}{6} \approx 67\%$
- 14. **C** First place is team C since they have 5 wins. Teams E and F are tied with 4 wins each after that. Since team F beat team E, we can determine that Team F is 2nd place and Team E is 3rd place.
- 15. **D** The total circumference traveled around the clock in an hour is $2\pi(9) = 18\pi$. So the distance traveled
 - in 33 minutes is $18\pi(\frac{33}{60}) = \frac{99\pi}{10}$
- 16. E The railroad will have 2 spikes every 3 feet and it is 5280(10) = 52800 feet long. So the total number of spikes needed can be found by $\frac{2}{3} = \frac{x}{52800} \Rightarrow x = 35200$. Now between the two people, 4 spikes will be nailed every minute. So there will be 4(60) = 240 every hour and 240(24) = 5760 every day. This means that it will take $\frac{35200}{5760} \approx 6.1$ days.

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17. **D** For convenience, we can place the vertex of the parabola at the origin, making the general equation $x^2 = 4py$ where p is the distance from the vertex to the focus. Knowing that the parabola has a width of 12 and a depth of 2 meters, we can determine that the "endpoints" of this section of the parabola are at (-6,2) and (6,2). Using the point (6,2), we can determine the distance:

$$x^{2} = 4 py \Longrightarrow (6)^{2} = 4 p(2) \Longrightarrow p = \frac{9}{2}$$

- 18. C Using the Intermediate Value Theorem, we can see that f(1) = -1 and f(2) = 8. Since the function is continuous and the function changes from negative to positive in this interval, there is a root between x = 1 and x = 2.
- 19. A The Circumcenter is the point that is equidistant from the three vertices of a triangle.

20. **B** 40 = 70 + (35 - 70)
$$e^{K(10)} \rightarrow \frac{6}{7} = e^{K(10)} \rightarrow \ln(\frac{6}{7}) = K(10) \rightarrow \frac{1}{10}\ln(\frac{6}{7}) = K$$

21. A Since there are 26 cards of each color there are 2 ways to choose 5 cards of a single color from the 26 available. So the probability is $\frac{2 \bullet_{26} C_5}{{}_{52}C_5}$. So j+l+n = 2+5+5 = 12.

22. **B**

 $P(x) = R(x) - C(x) = x(200 - x) - (2x^{2} + 80x + 900) = 200x - x^{2} - 4x^{2} - 80x - 900 = -3x^{2} + 120x - 900$ So the first zero profit: $-3x^{2} + 120x - 900 = 0 \rightarrow 3x^{2} - 120x + 900 = 0 \rightarrow 3(x^{2} - 40x + 300) = 0$ $3(x - 10)(x - 30) = 0 \rightarrow x = 10,30$. So the first zero profit is at 10 bags.

- 23. **E** The maximum profit occurs between the two zero profits. So the maximum occurs at x = 20.
- 24. **C** C is the only route that represents an Euler Circuit. One possible path is to start at the upper right corner, go across the top, travel along the upper of the two "v's", then travel back across the other "v", then follow around the outside of the rest of the rectangle, ending at the same point you started at.
- 25. **B** The distance that she travels for every revolution of the wheel is the circumference of the tire or 24π . Since she travels 60 revolutions per minute, her distance every minute is $24\pi(60)$. This means every hour her distance is $24\pi(60)(60)$. So if she is traveling ³/₄ of a day (or 18 hours), her distance is

 $24\pi(60)(60)(18) = 2 \cdot 2 \cdot 2 \cdot 3 \cdot \pi \cdot 2 \cdot 2 \cdot 3 \cdot 5 \cdot 2 \cdot 3 \cdot 5 \cdot 2 \cdot 3 \cdot 3 = 2^8 \cdot 3^5 \cdot 5^2 \cdot \pi$

26. **E**
$$P(Marshall) = (\frac{5}{6})(\frac{5}{6})(\frac{1}{6}) + (\frac{5}{6})(\frac{5}{6})(\frac{5}{6})(\frac{5}{6})(\frac{5}{6})(\frac{1}{6}) + \dots = \frac{(\frac{5}{6})(\frac{5}{6})(\frac{1}{6})}{1 - (\frac{5}{6})(\frac{5}{6})(\frac{5}{6})} = \frac{25}{91}$$

27. E
$$\frac{180(n-2)}{n} = 160 \rightarrow 180n - 360 = 160n \rightarrow n = 18$$

28. A Knowing that 7 people have never left the US we can determine that 27 have traveled to the abroad locations. Knowing that 2 people have been to all 3 countries and 11 have been to India and Poland, we can determine that 11 of the 15 that have been to India are accounted for. Also, using the information that 26 have been to at least one of Poland and Trinidad and Tobago, we can determine that the 27th person has only gone to India – now we have 12 of the 15 accounted for. We now know that since India has 15 people, 3 must have traveled only to India and Trinidad & Tobago. See the diagram below.

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We can now determine that the number who have been to only Poland and Trinidad and Tobago is 1 since the sum of visiting only 2 countries is 13. This then is our answer. The final Venn Diagram will be:



- 29. **D** $\frac{6}{15} = \frac{8}{x} \to 6x = 120 \to x = 20$
- 30. A Since the tests are all independent of one another, the probability that the next test will contain a bouncing ball problem is 99% or $\frac{99}{100}$