- 1. **B**.The difference between the 2^{nd} and 5^{th} angle is 3 times the common difference which means d=12. The angles average to 720/6=120. 120 5/2 (12) = 90.
- 2. **D**. Each side of the square will be π for and area of π^2 , whereas, the radius of the circle is 2 and area equals 4π , so the ratio is $4:\pi$
- 3. **C.** Since $\overline{DE} = \overline{CE} = \overline{CD}$, triangle CDE is equilateral and $\angle DCE$ is 60 degrees. Therefore the length of the arc DE is $1 - \pi$

 $\frac{1}{6}(2\pi)=\frac{\pi}{3}.$

- 4. A. Each interior angle of the pentagon has measure 108 degrees. Triangle ADE is isosceles with one angle 108 and two angles 36. This is also true for triangle ABC. The leaves 108-36-36=36 degrees for ∠CDA.
- 5. **C**. There are two equations a+10+c=35 and $a^2 + 100 = c^2$. The first yields c+a=25 and the second $c^2-a^2=(c+a)(c-a)=100$. Dividing c-a=4, or a=21/2, c=29/2.
- 6. **B.** From each of the circles, drop a perpendicular down from the center to one of the lines. Now draw a line from the line intersection to the center of the larger circle. These forms two 30-60-90 triangles. Let r is the radius of the large circle and x the smaller circle. In a 30-60-90 the hypotenuse is twice the short side. So, 2r=r+x+2x or x=r/3 = 3/3 = 1.
- 7. **D**. If the interior is 170, the exterior angles are 10 degrees. Since the sum of the exterior angles is 360 degrees, there must be 36 angles and sides.
- 8. **B**. \overline{CE} will be 3 so angle CEA is right. This means that \overline{AB} is a diameter. Also, \overline{CE} x $\overline{ED} = \overline{AE}$ x \overline{EB} so $\overline{EB} = 9/4$ and $\overline{AB} =$ 4+9/4 = 25/4

- 9. **C.** Let the base of the triangle be one of the sides of length 2. The side of length two can form any angle with the base between 0 and 180 degrees. To maximize the height of the triangle, take 90 degrees. The third side or hypotenuse is then $2\sqrt{2}$.
- 10. **B.** Let A be the point (0,0) and B (3,0). The shape is composed of (x,y) so that $x^{2} + y^{2} = 4((x-3)^{2} + y^{2})$ or $(x-4)^{2} + y^{2} = 4$, a circle of radius 2.
- 11. E. Draw lines from the center of the circle to the endpoints of the chord. From B, draw the tangent to the circle intersecting AC. This gives a quadrilateral with two 90 degree angles and one 80 degree angle. The last angle must be 100. The other two angles of this triangle are equal and must be 40 degrees.
- 12. A. The diagonal of the first square is the same as the diameter of the first circle which equal 2. The side length of the square is then $\sqrt{2}$ which is the diameter of the second circle. The area of the

second circle is then
$$\pi \left(\frac{\sqrt{2}}{2}\right)^2 = \frac{\pi}{2}$$
, one-half the area of the first one. The total area is then

$$\pi(1+\frac{1}{2}+\frac{1}{4}+\cdots)=2\pi$$

- 13. **C.** The exterior angle of a regular hexagon is 240 degrees, 2/3 of a full circle. The goat will be able to access $\frac{2}{3}(\pi 20^2)$ before wrapping around the next side. On either side, the goat can get to 60 degrees of a circle of radius 10 so $\frac{1}{3}(\pi 10^2)$, adding these one gets 300π
- 14. C. Since A is 90 degrees \overline{BD} is a diameter and equal to 25. Angle C is also 90 degrees. There are two distinct Pythagorean triples with hypotenuse 25, namely, 7-24-25 and 15-20-25. The perimeter is then 7+24+15+20=66.

- 15. A. This is a right triangle with hypotenuse
 10. If we connect the midpoint of the hypotenuse to the midpoints of the legs, these lines are perpendicular bisectors of the legs. Thus everything in the upper triangle will be closer to A and it can be seen to be ¹/₄ of the total area.
- 16. C. Need to find the side length of the smaller hexagon. ABC will be a 30-30-120 triangle and so is ABY. AC is $\sqrt{3}$ and XY is 1/3 of that. The



area of the hexagon is then 6

- 17. **D.**Draw lines from the center of the circle to each of the vertices and consider the quadrilateral formed be two consecutive triangles. The angle from the center is 60 degrees and the sides are both 1. So one diagonal is 1 because it forms an equilateral triangle and the other is 1 as it is a radius. The diagonals are perpendicular so the area is $\frac{1}{2}(1x1)=1/2$. There are 6 such quadrilaterals for a total area of 3.
- 18. **B.** Construct the semicircle centered at (2,0) and going through the origin and the point (4,0). If E is on the semicircle, angle AED will be right. If E is inside, it will be obtuse. The probability is then the area of the semicircle $\frac{1}{2}\pi(2)^2$ divided by the area of the

trapezoid, 15; or $\frac{2\pi}{15}$.

19. C. Since triangle AFD is isosceles and the continuation of EF would be the altitude, BE=EC=16. If EF=x, the altitude is 32-x and Pythagoras gives us: $16^2 + (32 - x)^2 = x^2$ and x=20. 20. **B**. The answer will be the radius of the

circumscribed circle. There is a formula: $R = \frac{abc}{4K}$

where K is the area of the triangle. Using a coordinate system, A at the origin (0,0), B at (14,0), C at (5, 12); the area is $\frac{1}{2}(14)(12)=84$. Carrying out the arithmetic, R=65/8. One can also find the coordinates of the circumcenter by finding the perpendicular bisectors of the sides. One is the line x=7 so the x coordinate will be 7. The y coordinate is found by equating the distance from (7,y) to two of the vertices. The distance can then be found using the distance formula.

- 21. **A**. Since BE is one-third of AB and BD is onethird of BC and they share angle B, triangle ADE is similar to triangle ABC and has 1/9 the area. Therefore the desired ratio is 1:8.
- 22. **D**. Use Heron's formula to get the area. The semiperimeter s=(7+8+9)/2=12 and the area: $A = \sqrt{s(s-a)(s-b(s-c))} = \sqrt{12(5)(4)(3)} = 12\sqrt{5}$ so $12\sqrt{5} = \frac{1}{2}9h$ or $h=\frac{8\sqrt{5}}{3}$
- 23. C. 12-35-37 is a right triangle so the diameter of the circumscribed circle is just the hypotenuse=37. The area of the triangle is 210 and the semiperimeter=42, so the radius of the inscribed circle is 5. The ratio is then 37/2 : 5 or 37:10.
- 24. **B**. The are 9(6)/2=27 total diagonals in the nonagon. Dividing the nonagon into three pieces (trapezoids) using the triangle, it is seen that there are 3 diagonals, 2 interior and the side of the triangle, that do not intersect the interior of the nonagon. 27-3(3)=18.
- 25. **A**. If we connect the centers of the larger circles, we get an equilateral triangle. The centroid of this triangle is the center of the smaller circle. The distance from the centroid, which is 2/3 the length of the median and height of the triangle, is equal to the sum of the radius of a big circle and small

circle.
$$\frac{2}{3} \left(\frac{6}{2} \sqrt{3} \right) = 3 + x$$
, so $x = 2\sqrt{3} - 3$

- 26. **B.** Put the triangle on a rectangular coordinate system with the right angle at the origin and other vertices at (5,0) and (0,12). The equation of the hypotenuse is y=12-(12/5)x and the angle bisector is the line y=x. They meet at the point (60/17, 60/17) which is $\frac{60}{17}\sqrt{2}$ from the origin.
- 27. C. Let the right triangle be ABC, point D be the intersection of altitude to the hypotenuse. By similar triangles 9/CD = CD/4, therefore CD = 6. Or, let x be the altitude and y and z be the legs of the triangle. From Pythagoras, $4^2 + x^2 = y^2$, $9^2 + x^2 = z^2$, adding these equations: $4^2 + x^2 + 9^2 + x^2 = y^2 + z^2 = (4+9)^2$, so x=6.
- 28. **B.** Draw lines from the center of the small circle to the diameters and from the center of the larger circle to the point of tangency of the two circles. If x is the radius of the small circle: $x\sqrt{2} + x = 1$, $x = \sqrt{2} 1$

- 29. A. Any 4 points chosen from the 7 will give one convex quadrilateral. 7 choose 4 = (7x6x5)/6=35.
- 30. **A**. Let the two legs have lengths 2x and 2y. The medians make two right triangles, therefore:

 $x^{2} + (2y)^{2} = 22^{2} = 484$ and $(2x)^{2} + y^{2} = 31^{2} = 961$. Adding these equations gives: $5x^{2} + 5y^{2} = 1445$, $x^{2} + y^{2} = 289$ so the length of the segment connecting the midpoints is 17. By similar triangles, the hypotenuse is 34.