1. C $\frac{3^{\frac{4}{5}}x^{\frac{-1}{3}}y^{\frac{-8}{9}}}{2^{-2}x^{\frac{5}{3}}y^{\frac{-2}{9}}}=\frac{2^2 3^{\frac{4}{5}}y^{\frac{2}{9}}}{x^{\frac{6}{3}}y^{\frac{8}{9}}}$

2. E $x^{\frac{2}{3}}-15x^{\frac{1}{3}}+44=0$ $\left(x^{\frac{1}{3}}-11\right)\left(x^{\frac{1}{3}}-4\right)=0$ $x^{\frac{1}{3}}-11=0$ or $x^{\frac{1}{3}}-4=0$ $x=1331$ or 64

$1331+64=1395$

3. B $2\log_8 3=\log_8 3^2$, $\frac{2}{\log_3 8}=\frac{\log_3 9}{\log_3 8}=\log_8 9$, $\frac{1}{3}\log_2 9=\log_{8^{\frac{1}{3}}} 9^{\frac{1}{3}}=\log_8 9$, but

$\frac{3}{2}\log_2 3=\log_2 9\sqrt{3}\neq\log_8 9$.

4. D $x^2-x-20=0$ or $x^2-x-\frac{11}{4}=1$. So, $x=-5,-\frac{3}{2},\frac{5}{2},$ or 4. Sum of solutions is 2.

5. B $\sqrt{20+x}=x$ $x^2-x-20=0$ $x=-4,5$ Checking for extraneous solutions, only $x=5$ works.

6. E A negative number to an irrational exponent cannot be real, but it is always complex.

7. B $\log_3\left(\log_{\frac{1}{2}} x\right)=2$ $\log_{\frac{1}{2}} x=9$ $x=\frac{1}{512}$

8. D $\log_4\frac{16}{9}+\log_3 2=\log_4 16-\log_4 9+\log_3 2$, which is also equivalent to $2-\log_2 3+\log_3 2$.

Substituting in for $\log_2 3$ and $\log_9 16$, we get $ab-a+\frac{1}{a}$.

9. A $2^{x-12}=2^{-x^2}$ $x^2+x-12=0$, so $x=-4$ or 3

10. C $216^{\log_{36} j}=6^{3\log_{36} j}$ $6^{\frac{3\log_6 j}{2}}=\left(6^{\log_6 j}\right)^{\frac{3}{2}}$, which is equivalent to $j^{\frac{3}{2}}$.

11. D 1 to any real power greater than zero is 1. The units digit of powers of 3 are repeatedly $3,9,7,1$, so the 2008th and 2009th powers of three have a units digit of 1 and 3 respectively. The units digit of powers of 7 are repeatedly $7,9,3,1$, so the 2010th power of 7 has a units digit of 9. Adding these four units digits ($1,1,3,9$) gives a sum with a units digit of 4.

12. B $\log 99.9^{256}$ $256\log 99.9=256\log 100+256\log .999$ After substituting the given information we get $512-.1024$. Thus, 99.9^{256} has 512 digits.

13. B $\prod_{n=2}^{10}\left(1+\log_{n!}(n+1)\right)=\prod_{n=2}^{10}\left(\log_{n!}(n+1)!\right)$ $\frac{\log 3!}{\log 2!}\frac{\log 4!}{\log 3!}\frac{\log 5!}{\log 4!}\cdots\frac{\log 11!}{\log 10!}=\log_{2!}11!$

14. A $6\left(6^7\right)=6^8$

15. B $e^{2x}+36=15e^x$ $e^{2x}-15e^x+36=0$ $\left(e^x-3\right)\left(e^x-12\right)=0$ $x=\ln 12,\ln 3$
$\ln 12-\ln 3=\ln 4$

16. C $y=\ln 4+3$ and $s=3-\ln 4$, so $y-s=\ln 4+3-(3-\ln 4)=\ln 4+\ln 4=\ln 16$

17. B $\log_4 16+\log_2 16=2+4=6$

18. B $\sum_{p=1}^{\infty}\frac{\log\left(n^p\right)}{n^p}=\sum_{p=1}^{\infty}\frac{p\log n}{n^p}=\log n\sum_{p=1}^{\infty}\frac{p}{n^p}$ Let $A=\sum_{p=1}^{\infty}\frac{p}{n^p}=\left(\frac{1}{n}+\frac{2}{n^2}+\frac{3}{n^3}\ldots\right)$.
$nA=\left(1+\frac{2}{n}+\frac{3}{n^2}+\frac{4}{n^3}\ldots\right)$. So,
$nA-A=\left(1+\frac{2}{n}+\frac{3}{n^2}+\ldots\right)-\left(\frac{1}{n}+\frac{2}{n^2}+\ldots\right)=\left(1+\frac{1}{n}+\frac{1}{n^2}+\ldots\right)=\sum_{p=0}^{\infty}\frac{1}{n^p}$. Since $nA-A=\sum_{p=0}^{\infty}\frac{1}{n^p}$
and $A=\sum_{p=1}^{\infty}\frac{p}{n^p}$, $\sum_{p=1}^{\infty}\frac{p}{n^p}=\frac{1}{(n-1)}\sum_{p=0}^{\infty}\frac{1}{n^p}$. Thus $\log n\sum_{p=1}^{\infty}\frac{p}{n^p}=\frac{\log n}{(n-1)}\sum_{p=0}^{\infty}\frac{1}{n^p}=\frac{n\log n}{(n-1)^2}$.

19. D The 3rd term is the constant term. $\left(\frac{6!}{4!3!}\right)\left(9x^2\right)^2\left(\frac{-1}{2x}\right)^4=(15)\left(81x^4\right)\left(\frac{1}{16x^4}\right)=\frac{1215}{16}$

20. B $x^{1+x^{1+x^{1+x\ldots}}}=2$ $x^{1+2}=2$ $x=\sqrt[3]{2}$

21. E The function $f(x)=\log x^2$ passes through all four quadrants.

22. D $i^{86}+411i^{75}-17i^{30}+2i^9=i^2+411i^3-17i^2+2i=-1-411i+17+2i=16-409i$

23. A Let $x=2\log_a b$. $x^8+\frac{1}{x^8}=47$. $\left(x^4+\frac{1}{x^4}\right)^2=49$, so $x^4+\frac{1}{x^4}$ =7. After repeating the process, $x^2+\frac{1}{x^2}=3$ and $x+\frac{1}{x}=\sqrt{5}$. $x^3+\frac{1}{x^3}=\left(x+\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}\right)-\left(x+\frac{1}{x}\right)$.

24. C $2.5<e<3$, so $\left(\frac{5}{2}\right)^5<e^5<3^5$. $\sqrt{1020}<32$. $243\log 1=0$. $\frac{e^6}{2}=\left(e^5\right)\left(\frac{e}{2}\right)$ and since $e>2.5$, $\frac{e^6}{2}>e^5$.

25. C $f(x)$ and $g(x)$ are inverses of each other. $4=\sqrt{x+2}$ $16=x+2$ $x=14$

26. A 1 to any real power is 1; therefore, $2\Delta\left(1^{(3\Delta 4)}\right)=2\Delta 1$. $2^{(1-2)}=\frac{1}{2}$

27. C $h(x)=2-\log_4\left(x-11\right)$ $x-11=0$ $x=11$

28. D $j(x)=x^{\frac{1}{3}}$. For $j(x)$ to be a positive integer less than 5, $0<x^{\frac{1}{3}}<5$. $0<x<125$ and x must be a perfect cube. So, $x=1,8,27,64$ $1+8+27+64=100$

29. A $\log_p q>1$ when $q>p$, so $a=\frac{1}{2}$. Since $\log_p q$ can never equal 1 and $a=\frac{1}{2}$, $b=\frac{1}{2}$.
$\frac{1}{4}-\frac{1}{4}=0$

30. B $f(2x)=\frac{\left(2^x-1\right)\left(2^x+1\right)}{2^x+1}=2^x-1$ $f(10)=2^5-1$ and $f(12)=2^6-1$
$2\log_{31}\left(2^5-1\right)-\log_{63}\left(2^6-1\right)$ $2\log_{31}31-\log_{63}63=2-1=1$