Polynomial and Rational Functions – Theta  Solutions  

C 1. Using synthetic division, find roots are 3, \(\frac{1}{2},-2\). Largest root is 3.

A 2. Factoring gives \(4\sqrt{1+x^2} - 3\sqrt{1+x^2}\) which is \(\sqrt{1+x^2}\).

C 3. When roots are equal, the discriminant \(= 0\).
\[p^2 - 4 \cdot 1 \cdot 2p = 0, \quad p^2 - 8p = 0, \quad p = 0, 8.\]
There are 2 distinct values for \(n\).

C 4. Find the vertex by completing the square:
\[y = -2(x^2 - 6x + 9) - 24 + 18.\] The vertex is \((3, -6)\). Since this parabola opens down, the range would be \((-\infty, -6]\).

A 5. \(f(0+1) = f(0)f(1) = -2;\)
\(f(0) = 1; f(1+1) = f(1)f(1) = 4;\)
\(f(2) = 4; f(1+2) = f(1)f(2) = -2 \cdot 4;\)
\(f(3) = -8\)

B 6. \(\frac{1}{x+2} - \frac{3}{x-1} + \frac{1}{x^2 + x - 2}\), factor \(x^2 + x - 2\) to find it is the common denominator.
\[\frac{x-1-3x-6+1}{x^2 + x - 2} = \frac{-2x-6}{x^2 + x - 2}\]

A 7. \(f(x) = -x^2 + 1, g(x,y) = x(1+y)\)
To find, \(g(f(2), 3) = f(2) = -3;\)
\(g(-3, 3) = -3(4) = -12\)

B 8. Since \(a\) is negative \(1-a\) is positive so use \((x-1)^2, (1-a-1)^2 = (-a)^2 = a^2\).

D 9. To find what must be done to make \(\frac{1}{x+3} = x\), we must find the inverse of \(\frac{1}{x+3}\)
then substitute this into \(\frac{1}{2-5x}\). So
\[x = \frac{1}{y+3}, x(y+3) = 1, y+3 = \frac{1}{x}, \quad y = \frac{1}{x} - 3.\]

This gives
\[\frac{1}{2-5\left(\frac{1}{x}-3\right)} = \frac{1}{2-\frac{5}{x}+15}\]
\[\frac{1}{17-\frac{5}{x}} = \frac{1}{17x-5} = \frac{x}{17x-5}.\]

A 10. Find the vertex of the parabola:
\[y = -2\left(x^2 - 2x + 1\right) - 1 + 2, \quad y = -2(x-1)^2 + 1\]
making the vertex \((1,1)\). Since the line passes through the origin \((0,0)\) the slope of the line is 1, which makes the equation of the line \(x = y\).

B 11. \(\frac{1}{a+b} = \frac{a+b}{ab} = \frac{(a+b)^2}{ab}\).

C 12. \(\sqrt{x+1} + \sqrt{x-1} = 3\left(\sqrt{x+1} - \sqrt{x-1}\right),\)
\[\sqrt{x+1} + \sqrt{x-1} = 3\sqrt{x+1} - 3\sqrt{x-1},\]
\[4\sqrt{x-1} = 2\sqrt{x+1},\]
\[16x - 16 = 4x + 4, 12x = 20, x = \frac{5}{3}.\]

D 13. We need to find the values for \(p\) and \(q\).
\[\frac{p}{q} = \pm 1, \pm 3, \pm 5, \pm 15 = \pm 1, \pm 2\]
\[\pm 1, \pm 3, \pm 5, \pm 15, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{5}{2}, \pm \frac{15}{2}\]
which is 16 POSSIBLE roots.

E 14. The line has a slope of 4 so the line perpendicular to this would have a slope of \(-\frac{1}{4}\) and containing \((-1,3)\). The equation would be \(x+4y = 11\).

A 15. Find the possible integer roots which would be \(\pm 1, \pm 3\). Using synthetic or substitution none of these satisfy the equation.
C 16. Set the denominator equal to zero.

B 17. Completing the square on the left hand sides gives \( \left( x^2 - 3x + \frac{9}{4} \right) + 2 - \frac{9}{4} = \left( x - \frac{3}{2} \right)^2 - \frac{1}{4} \)
so the value of \( p \) is \( \frac{1}{4} \).

C 18. Doing synthetic division with \(-3\) gives the new equation as \( 2x^3 - 9x^2 + 14x - 5 = 0 \).

The new possible roots are now \( \pm 1, \pm 5, \pm \frac{1}{2}, \pm \frac{5}{2} \).

Trying synthetic division gives \( \frac{1}{2} \) as a root.
From the original equation, the sum of the roots is \( \frac{3}{2} \). The sum of the real roots is \( -3 + \frac{1}{2} = -\frac{5}{2} \).
So subtracting that from the sum of the roots gives the sum of the imaginary roots as \( 4 \).

E 19. If \( 5 \) is a root, \( x - 5 \) is a factor. Doing division with this makes the remainder 0.

A 20. \( f(x) \) must be positive. Factoring the expression under the root gives \( \sqrt{x(x-1)(x+1)} \). The critical values for \( x \) are 0, 1, -1. Put these on a number line and test the zones. This gives \([-1,0] \cup [1, \infty) \).

D 21. Switch \( x \) and \( y \) and solve for \( y \).
\[
x = \frac{1}{y+3},\quad y(x+3) = 1,\quad y+3 = \frac{1}{x},\quad y = \frac{1}{x} - 3
\]

B 22. The remainder when \( y^2 + 2y + 4s \) is divided by \( y-1 \) is \( 4s - 2 \). The remainder when \( y^2 + sy + 2s^2 \) is divided by \( y-1 \) is \( 2s^2 + s + 1 \).

Set these two remainders equal and solve for \( s \).
\[
2s^2 + s + 1 = 4s - 2,\quad 2s^2 - 3s + 3 = 0.
\]
The sum of the roots is \( \frac{-b}{a} = \frac{3}{2} \) which is \( 1 \frac{1}{2} \).

B 23. \( f(g(x)) = 2^{\log_2 x} \) which equals \( x \).

C 24. \( (x-4)(x-5) = x^2 - 9x + 20, A = -9, B = 20 \)
\[
(x-2)(x-9) = x^2 - 11x + 18, C = -11, D = 18
\]
x^2 - 9x + 18 = (x-6)(x-3) so the roots are 6 or 3.

C 25. \( g(2) = -1, f(-1) = 4 \).

C 26. Using the information in the problem, the points on the graph are \((-2,0),(1,0),(0,0)\).

Substituting, we get three equations:
\[
\begin{align*}
0 &= -8 + 4a - 2b + c \\
0 &= 1 + a + b + c & \text{making } c = 0. \\
0 &= 0 + 0 + 0 + c
\end{align*}
\]
Substitute this into the first two equations to get
\[
\begin{align*}
4a - 2b &= 8 \\
a + b &= -1
\end{align*}
\]
Solve this system to get \( a = 1, b = -2 \). So the original equation should now be \( P(x) = x^3 + x^2 - 2x, P(-1) = 2 / \)

C 27. \( 8^{\frac{2}{3}} = \frac{1}{4} \)

B 28. Let \( g(x) = y \). Substituting \( y \) into \( f(x) \), we get \( y - 3 \). Since this is \((f \circ g)(x)\), \( y - 3 = x^2 + 1, y = x^2 + 4 \) which is \( g(x) \).

D 29. Square both sides and put in standard form gives a hyperbola. \( 1 = \frac{y^2}{4} - \frac{x^2}{16} \). Graphing by hand shows that it is the lower half.

D 30. Complete the square to find the vertex. The maximum value would be the y-coordinate of the vertex. \( y = -\left( x^2 + 5x + \frac{25}{4} \right) + 2 + \frac{25}{4} \).
\[
2 + \frac{25}{4} = \frac{33}{4}.
\]