For all questions, answer choice "E) NOTA" means none of the above answers is correct.

1. Evaluate $2\cos\left(\frac{11\pi}{12}\right)$.

A) $\sqrt{2+\sqrt{3}}$ B) $-\sqrt{2+\sqrt{3}}$ C) $-\sqrt{2-\sqrt{3}}$ D) $\sqrt{2-\sqrt{3}}$ E) NOTA

2. Evaluate $\lim_{h\to 0}\frac{(1+h)^3-2(1+h)^2+1-1^3+2(1)^2-1}{h}$.

A) 1 B) -1 C) 0 D) Does not exist E) NOTA

3. Determine the non-degenerate conic section represented by the equation $\begin{vmatrix} y & x & -y \\ 5 & 1 & -2 \\ x & y & x \end{vmatrix} = -5$.

A) circle B) hyperbola C) ellipse D) parabola E) NOTA

4. Using the conic section defined in the previous question, let θ be the angle of rotation of the axes. Evaluate $\sin^{-1}(\tan 2\theta)$.

A) $-\pi/6$ B) $-\pi/3$ C) $2\pi/3$ D) $5\pi/6$ E) NOTA

Use the following table to answer questions 5-6.

x	-3	-2	-1	0	1	2	3
$f(x)$	0	-1	3	2	-2	1	-3

5. Let $f^n(x)$ represent the concatenations of $f(x)$ n times (ex. $f^3(x)=f(f(f(x)))$), and let $g^n(x)$ represent the concatenations of $f^{-1}(x)$ n times (ex. $g^2(x)=f^{-1}(f^{-1}(x))$). Evaluate $f^6(0)\cdot g^4(2)$.

A) -3 B) 0 C) 2 D) 3 E) NOTA

6. Find the average rate of change of $f^{-1}(x)$ on the interval $[0,3]$, assuming f is continuous and one-to-one.

A) $3/2$ B) $-5/3$ C) $-3/5$ D) $2/3$ E) NOTA

7. The convex base of a pyramid is formed by the points $(2,0,1)$, $(-1,3,1)$, $(0,7,1)$, $(4,11,1)$, $(3,14,1)$, and $(5,5,1)$, with all of the vertices of the hexagonal base meeting at a common vertex located at the point $(4,6,10)$. What is the volume of the solid?

A) 120 B) 135 C) $400/3$ D) 150 E) NOTA

8. Simplify $\left(\left(4\text{cis}25^\circ\right)\left(\text{cis}20^\circ\right)\right)^3$, where $\text{cis}\theta = \cos\theta + i\sin\theta$.

A) $32+32i\sqrt{3}$ B) $32\sqrt{2}-32i\sqrt{2}$ C) $-32\sqrt{2}+32i\sqrt{2}$ D) $-32\sqrt{2}-32i\sqrt{2}$ E) NOTA

9. The sides of a triangle have lengths 21, 10, and 17. Find the length of the altitude to the longest side of this triangle.

A) 8.5 B) 8 C) 6 D) 5 E) NOTA

10. Evaluate $\cos\left(\sin^{-1}\left(\frac{7}{25}\right)+\tan^{-1}\left(\frac{15}{8}\right)\right)$.

A) $87/425$ B) $416/425$ C) $608/425$ D) $783/425$ E) NOTA

11. A family wants to have three children. What is the probability that the family will have three girls, given that one of the children will be a girl?

A) $1/8$ B) $1/4$ C) $7/8$ D) $1/7$ E) NOTA

12. What is the remainder when $f(x)=x^{2011}-9x^{2010}+4x^{2008}+17x^{1066}-3x^{211}+x^2+5x+7$ is divided by $x+1$?

A) 0 B) 9 C) 15 D) 23 E) NOTA

13. This mathematician is credited with developing a well-renowned proof of the Fundamental Theorem of Algebra. He is also known for publishing his ideas for a complex coordinate plane. Who is this mathematician?

A) Nikolaus Bernoulli B) Carl Friedrich Gauss C) Jean-Robert Argand
D) Leonhard Euler E) NOTA

14. Find the sum of all values of θ on the interval $[-\pi,\pi]$ that satisfy the equation $4\sin^4\theta+2\sin^3\theta-4\sin^2\theta-\sin\theta+1=0$.

A) 0 B) $\pi/2$ C) $3\pi/2$ D) $5\pi/2$ E) NOTA

15. What is the vertex of the parabola defined by the parametric equations $x=t^2+t$ and $y=2t-1$?

A) $\left(1/4,2\right)$ B) $\left(2,1/4\right)$ C) $\left(-2,-1/4\right)$ D) $\left(-1/4,-2\right)$ E) NOTA

16. Evaluate the infinite sum $\frac{3}{4}+\frac{1}{2}+\frac{9}{16}+\frac{1}{4}+\frac{27}{64}+\frac{1}{8}+\ldots$, where the sequences consisting of every other term are both geometric.

A) 4 B) 5 C) $9/4$ D) series diverges E) NOTA

17. What is the domain of the function $f(x)=\log_x\left(6x^3-31x^2+34x+15\right)$?

A) $\left(-\frac{1}{3},\frac{5}{2}\right)\cup(3,\infty)$ B) $\left(0,\frac{5}{2}\right)\cup(3,\infty)$ C) $\left(-\frac{1}{3},0\right)\cup\left(0,\frac{5}{2}\right)\cup(3,\infty)$
D) $(0,1)\cup\left(1,\frac{5}{2}\right)\cup(3,\infty)$ E) NOTA

18. What is the units' digit of $(23)^{23}(16)^{16}-(8)^8(12)^{12}$?

A) 2 B) 4 C) 6 D) 8 E) NOTA

19. What is the greatest common factor of 2011_9 and 4022_5?

A) 2011_3 B) 100_2 C) 10_2 D) 13_5 E) NOTA

20. Which of the following statements are true, given 3-dimensional vectors $\vec{a}$ and $\vec{b}$ and 3×3 matrices A and B?

I. $\vec{a}\times\vec{b}=\vec{b}\times\vec{a}$ II. $\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{a}$ III. $(AB)^T=B^TA^T$ IV. $\vec{a}\cdot\vec{a}=\left(\vec{a}\right)^2$ V. $\vec{a}\times\left(\vec{b}\times\vec{c}\right)=\left(\vec{a}\times\vec{b}\right)\times\vec{c}$

A) I, II, & III B) II, III, & V C) I, II, III, & IV D) all are true E) NOTA

21. I love pizza, so I contact my local Pizza Pi to order a pizza. Thankfully, Pizza Pi has lots of toppings to offer, plus they offer a special crust deal. Pizza Pi has 10 toppings, with a choice of three different toppings per pizza. Pizza Pi's special crust deal consists of each individual slice of pizza having its own specially flavored crust. Assuming the pizza is a perfect disk, I always want three toppings on my pizza, and each of the eight slices has a different flavored crust, how many unique pizzas can I order?

A) $\frac{10!}{3!}$ B) $\frac{4\cdot10!}{3}$ C) $10!$ D) $8\cdot10!$ E) NOTA

22. Find a vector that is orthogonal to both vectors $\vec{v}$ and $\vec{w}$. $\vec{v}$ is orthogonal to both $\vec{a}=3\vec{i}+2\vec{j}-\vec{k}$ and $\vec{b}=-\vec{i}+4\vec{j}-\vec{k}$, and $\vec{w}$ is orthogonal to both $\vec{c}=7\vec{i}-2\vec{j}+5\vec{k}$ and $\vec{d}=-\vec{i}+\vec{j}+\vec{k}$. In essence, evaluate $\vec{v}\times\vec{w}$, which is equal to $\left(\vec{a}\times\vec{b}\right)\times\left(\vec{c}\times\vec{d}\right)$.

A) $-7\vec{i}-12\vec{j}+5\vec{k}$ B) $37\vec{i}+27\vec{j}-13\vec{k}$ C) $47\vec{i}+27\vec{j}+\vec{k}$ D) $188\vec{i}-108\vec{j}+4\vec{k}$ E) NOTA

23. Simplify $\frac{3-5i}{i+1}-\frac{7-i}{2-3i}$, where $i=\sqrt{-1}$.

A) $-\frac{12}{17}+\frac{20}{17}i$ B) $-\frac{4}{13}+\frac{33}{13}i$ C) $-\frac{30}{13}-\frac{71}{13}i$ D) $\frac{4}{13}-\frac{33}{13}i$ E) NOTA

24. The solution to the equation $2^{x+2}=3^{2x+1}$ can be expressed in the form $x=\log_{4.5}b$. Evaluate $\frac{9}{2b}$.

A) 1 B) $8/27$ C) 4 D) $27/8$ E) NOTA

25. Let $f(x)=2x^4-13x^3+7x^2-9x-3$. What is the positive difference between the sum of the reciprocals of the roots of $f(x)$ and the sum of the squares of the roots of $f(x)$?

A) $7/4$ B) $9/4$ C) $19/4$ D) $153/4$ E) NOTA

26. Given that $a=\log 2$ and $b=\log 3$, expand $\log x$, where $x=$ the number of distinct 6-letter permutations of the letters in the word DALLAS.

A) $a+2b+1$ B) $2ab$ C) $a+b^2+1$ D) $3a+2b+1$ E) NOTA

27. A hyperbola is defined by the equation $9x^2-16y^2+18x+64y-199=0$. What is the shortest distance from a focus of the hyperbola to either of its asymptotes?

A) 3 B) 5 C) $15/4$ D) $23/4$ E) NOTA

28. You're having so much fun taking this test, but you've lost track of the time! It is currently 10:30 a.m., and you have only a few minutes left to finish this test. Unfortunately, you become mesmerized with the time and can only focus on the clock. You notice that the minute hand is 4 inches long and that the hour hand is 2 inches long. Instead of finishing the test, you wonder, what is the distance between the tip of the minute hand and the tip of the hour hand, in inches?

A) $2\sqrt{7}$ B) $2\sqrt{5-2\sqrt{2}}$ C) $2\sqrt{5+2\sqrt{2}}$ D) $2\sqrt{5+2\sqrt{3}}$ E) NOTA

29. What is the sum of the rank and trace of the matrix $A=\begin{bmatrix}1 & 2 & 0 & 3\\ 1 & -2 & 3 & 0\\ 0 & 0 & 4 & 8\\ 2 & 4 & 0 & 6\end{bmatrix}$?

A) 11 B) 12 C) 13 D) -45 E) NOTA

30. Solve the inequality $\dfrac{3x-2}{2x+1}\geq\dfrac{x-1}{x+3}$.

A) $\left(-\infty,-4-\sqrt{21}\right]\cup\left[-4+\sqrt{21},\infty\right)$ B) $\left(-\infty,-4-\sqrt{21}\right]\cup\left(-3,-\dfrac{1}{2}\right)\cup\left[-4+\sqrt{21},\infty\right)$

C) $\left[-4-\sqrt{21},-3\right)\cup\left(-\dfrac{1}{2},-4+\sqrt{21}\right]$ D) $\left(-\infty,-4-\sqrt{21}\right]\cup(-3,\infty)$

E) NOTA