For all questions, answer choice "E) NOTA" means none of the above answers is correct.

1. Evaluate: \( \int_{-2}^{3} (4x^3 + 5x^2 + 6x + 7) \, dx \)

   A) 110 \hspace{1cm} B) \frac{350}{3} \hspace{1cm} C) \frac{428}{3} \hspace{1cm} D) \frac{520}{3} \hspace{1cm} E) NOTA

2. \( A \) is a \( 3 \times 3 \) matrix such that \( |A| = 4 \). Find the value of \( |A^T| \cdot |2A| \).

   A) 48 \hspace{1cm} B) 128 \hspace{1cm} C) 32 \hspace{1cm} D) 8 \hspace{1cm} E) NOTA

3. The Frisco Community Center is draining its lap pool for a midsummer cleaning. The pool is a rectangular prism measuring 50 yards long by 20 yards wide by 4 feet deep. The volume of water is decreasing at a constant rate of 100 yd\(^3\) /min. At what rate is the height of the water decreasing, in yd/min?

   A) \( \frac{1}{10} \) \hspace{1cm} B) \( \frac{3}{10} \) \hspace{1cm} C) \( \frac{1}{40} \) \hspace{1cm} D) \( \frac{3}{40} \) \hspace{1cm} E) NOTA

4. If \( A \equiv 5 \pmod{6} \), which of the following could be equal to \( A \)?

   A) 121 \hspace{1cm} B) 255 \hspace{1cm} C) 719 \hspace{1cm} D) 1023 \hspace{1cm} E) NOTA

5. Find the value of \( \int_{\arcsin \frac{1}{4}}^{\arcsin \frac{1}{2}} \left( \frac{\sin x}{e^{\sin x}} + \frac{d(\arcsin y)}{dy} \right) + \ln \left( \frac{e^2 - 1}{e - 1} - 1 \right)^3 \).

   A) \( \frac{37}{6} \) \hspace{1cm} B) \( \frac{34 + 3\sqrt{3}}{6} \) \hspace{1cm} C) \( \frac{17}{6} \) \hspace{1cm} D) \( \frac{14 + 3\sqrt{3}}{6} \) \hspace{1cm} E) NOTA

6. Determine the sum of all real solutions to \( x^5 + 5x(x^3 + 1) + 10x^2(x + 1) - 14 = 228 \).

   A) \(-1\) \hspace{1cm} B) \(-5\) \hspace{1cm} C) 1 \hspace{1cm} D) 2 \hspace{1cm} E) NOTA

7. What is the \( n \)th derivative of the function \( f(x) = -\frac{1}{x} \)?

   A) \( \frac{n!}{x^{n+1}} \) \hspace{1cm} B) \( \frac{(-1)^n n!}{x^n} \) \hspace{1cm} C) \( \frac{(-1)^{n-1} n!}{x^{n-1}} \) \hspace{1cm} D) \( \frac{(-1)^n n!}{x^{n+1}} \) \hspace{1cm} E) NOTA
8. If \((a, b, c)\) satisfies the system \[
\begin{align*}
2x + 4y + 7z &= 81 \\
7x + 2y + 4z &= 58 \\
4x + 7y + 2z &= 30
\end{align*}
\]
find the value of \(a + b + c\).

A) 13  B) 10  C) −8  D) −3  E) NOTA

9. Nissan produces \(C\) cars per day depending on the number of hours, \(H\), the factor operates on that day as described by the function \(C = H\sqrt{H^2 - 144}\). The factory randomly operates between 13 and 15 hours each day. What is the average number of cars the factory produces in a given day, rounded to the nearest whole number?

A) 201  B) 200  C) 101  D) 100  E) NOTA

10. Given the recursive progression \(d_{n+1} = \frac{d_n^2 + a l^2 + as}{d_n}\) for \(0 < a, s, l < 1\), where \(d_1 = 1\), find the value of \(\lim_{n \to \infty} d_n\), given that such a limit exists.

A) \(\sqrt{\frac{a l^2 - 1}{-as}}\)  B) \(\sqrt{\frac{as}{al^2 - 1}}\)  C) \(\sqrt{\frac{al^2 - 1}{as}}\)  D) \(\sqrt{\frac{-as}{al^2 - 1}}\)  E) NOTA

11. How many triangles are in the figure, given that the edges of the triangles must be edges in the figure?

A) 26  B) 28  C) 19  D) 20  E) NOTA

12. Find the volume when the region bounded by \(x - 3y = -18\), \(x + 3y = -18\), \(x = -24\), and \(x = 3\) is rotated about the \(x\)-axis.

A) \(351\pi\)  B) \(1053\pi\)  C) \(702\pi\)  D) \(1296\pi\)  E) NOTA

13. 2000 math team students went to Six Flags this week. 24% of the students who rode Superman also rode Batman. 15% of students who rode Batman also rode Superman. 33 students rode the Mini Mine Train. No student rode Batman and/or Superman and also the Mini Mine Train. 1% of students didn’t ride any of the three rides. How many students rode Batman?

A) 750  B) 825  C) 1260  D) 1320  E) NOTA
14. \[ \sum_{x=1}^{501} \frac{1}{x^2 + 3x + 2} = \frac{m}{n} , \text{ where } m \text{ and } n \text{ are relatively prime positive integers. Find } m-n. \]

A) 126 \quad B) 505 \quad C) -126 \quad D) -505 \quad E) NOTA

15. Evaluate: \[ \int_{0.5}^{1} \frac{(\sin^2 x + \cos^2 x)(1 + \cot^2 x) \tan x}{(1 + \tan^2 x) \cot x} \, dx \]

A) \sin 1 - \sin 0.5 \quad B) \tan 1 - \tan 0.5 \quad C) 0.5 \quad D) 1.5 \quad E) NOTA

16. Let \( X \) be a continuous random variable with probability density function
\[ f(x) = \begin{cases} Cx^2 e^{-2x^3}, & x > 0 \\ 0, & x \leq 0 \end{cases} \]
What is the value of \( C \)?

A) 3 \quad B) 6 \quad C) -3 \quad D) -6 \quad E) NOTA

17. Find the area enclosed by \( y = 6x^2 + 9x - 4 \) and \( y = 5x + 6 \).

A) \frac{188}{27} \quad B) \frac{512}{27} \quad C) \frac{188}{9} \quad D) \frac{512}{9} \quad E) NOTA

18. Evaluate: \( \lim_{x \to \infty} \left( \sqrt{2x^2 + 7x} - \sqrt{2x^2 + 3x} \right) \)

A) 2 \quad B) \infty \quad C) 0 \quad D) 4 \quad E) NOTA

19. The Dallas Nissan office is test-driving the 2012 Altima. Until it reaches 40 yds/hr, the car accelerates at a rate of \( 12t + 4 \) yds/hr^2, where \( t \) represents the time, in hours, that has passed since the car began accelerating, then stops accelerating. From past tests, Nissan knows the constant term of the velocity function is +8. How many yards does the car travels before it stops accelerating?

A) 40 \quad B) 88 \quad C) 2 \quad D) 3 \quad E) NOTA

20. Evaluate: \( \int_{1}^{2} \int_{0}^{1.5} \frac{x}{\sqrt{9-y^2}} \, dy \, dx \)

A) \frac{1}{12} \ln \frac{2}{5} \quad B) \sin^{-1} \frac{1}{4} \quad C) \frac{\pi}{4} \quad D) \frac{1}{12} \ln \frac{5}{2} \quad E) NOTA
21. Find the value of \( \frac{dy}{d(x^{-1})} \), given that \( y = \frac{\ln x}{3x^2} \).

A) \( \frac{1+2\ln x}{3x} \)  
B) \( -\frac{1+2\ln x}{3x} \)  
C) \( \frac{1-2\ln x}{3x} \)  
D) \( \frac{2\ln x - 1}{3x} \)  
E) NOTA

22. Evaluate: \( \lim_{x \to 0} \frac{\csc^2 x (\cos x - 1) \tan x}{x \sec x} \)

A) 0  
B) 1  
C) \( \pi \)  
D) \( \frac{1}{2} \)  
E) NOTA

23. The velocity \( \frac{dL}{dt} \) of a runaway train equals \( \frac{t+1}{L} \), where \( t \) represents the amount of time, in min, that has passed since the train got away, and \( L \) represents the distance, in mi, the train has traveled since it got away. The train must be stopped before it reaches a town 11 miles away from where the train originally got away. How many minutes do the officials have left if one minute has passed and the train already traveled 2 miles?

A) 10  
B) 12  
C) 9  
D) 7  
E) NOTA

24. Find the area bounded by the directrices of the conic sections with equations

\( \frac{(y-2)^2}{25} + \frac{(x+4)^2}{9} = 1 \) and \( \frac{(x+1)^2}{9} - \frac{(y-2)^2}{16} = 1 \).

A) 80  
B) 45  
C) 2.25  
D) 60  
E) NOTA

25. The revenue generated by producing \( n \) pencils is \( -\frac{1}{4}n^2 + 6n \) while the cost of producing those pencils is \( \ln(n+1) + 1000 \). How many pencils, to the nearest whole number, must be produced in order to maximize the profit in manufacturing those pencils?

A) 1  
B) 10  
C) 12  
D) 24  
E) NOTA

26. Use Newton's Method to find the third approximation of a real solution to the equation \( 2x^3 - 4x + 1 = 0 \), beginning with a first approximation of \( x_i = 0 \).

A) \( \frac{1}{4} \)  
B) \( \frac{7}{29} \)  
C) \( \frac{7}{32} \)  
D) \( \frac{15}{58} \)  
E) NOTA
27. What is the 2011th derivative of \( f(x) = -\cos x \)?

A) \( \cos x \)  
B) \( -\cos x \)  
C) \( \sin x \)  
D) \( -\sin x \)  
E) NOTA

28. Given \( \frac{dy}{dx} = \frac{\sin x + e^x}{6y + 2} \), \( y(0) = \frac{4}{3} \), which of the following represents the relationship between \( x \) and \( y \)?

A) \( 8 - \cos x + 2y + e^x + 3y^2 = 0 \)  
B) \( 8 - \cos x - 2y + e^x - 3y^2 = 0 \)  
C) \( 6 - \cos x + 2y + e^x + 3y^2 = 0 \)  
D) \( 8 - \cos x - 2y + e^x + 3y^2 = 0 \)  
E) NOTA

29. For \( 7x^3 y^2 + 3x + 4y^3 + 5x^2 = 66 \), evaluate \( \frac{dy}{dx} \) at the point \((2, -1)\).

A) \( \frac{101}{106} \)  
B) \( -\frac{101}{106} \)  
C) \( \frac{302}{83} \)  
D) \( -\frac{302}{83} \)  
E) NOTA

30. Dave, a Dallas native who now works on Wall Street, has developed a formula to predict the average daily price \( P \) of the stock of a sports drink manufacturing company based on the number of athletic shoes sold the day before, denoted \( S \), and the average temperature that day in degrees Fahrenheit, denoted \( F \). His formula is \( P(S, F) = F^4 \sqrt{S} - 4F^3 (.04S)^{1.5} \) and only works for positively-valued temperatures. Dave only wants to buy the stock when its average daily price is increasing. If 2500 athletic shoes were sold yesterday, then what is the most inclusive range of temperature, in degrees Fahrenheit, the average temperature must be today in order for Dave to purchase the stock?

A) \( F > 60^\circ \)  
B) \( F > 70^\circ \)  
C) \( F > 80^\circ \)  
D) \( F > 90^\circ \)  
E) NOTA