Answers:

1. D
2. B
3. A
4. C
5. A
6. D
7. E
8. A
9. C
10. D
11. E
12. A
13. D
14. D
15. C
16. B
17. B
18. E
19. A
20. C
21. D
22. E
23. C
24. B
25. C
26. D
27. D
28. B
29. A
30. A

Solutions:

1. $\int_{-2}^{3}\left(4x^3+5x^2+6x+7\right)dx=\left(x^4+\frac{5}{3}x^3+3x^2+7x\right)\Big|_{-2}^{3}=\left(81+45+27+21\right)$
$-\left(16-\frac{40}{3}+12-14\right)=\frac{520}{3}$

2. $\left|A^T\right|\cdot\left|2A\right|=4\left(2^3\cdot 4\right)=128$

3. $V=1000h\Rightarrow\frac{dV}{dt}=1000\frac{dh}{dt}\Rightarrow\frac{dh}{dt}=-100/1000=-1/10$, so the water is decreasing at $1/10$ yd/min

4. A is 1 less than a multiple of 6. The only one of the answer choices that fits that criterion is 719.

5. $\frac{i^{473}}{i^{437}}+\frac{d\left(\sin x\right)}{dx}\Big|_{x=\pi/3}+\frac{d\left(\arcsin y\right)}{dy}\Big|_{y=4/5}+\ln\left(\frac{e^2-1}{e-1}-1\right)^3=1+\cos\frac{\pi}{3}+\frac{1}{\sqrt{1-\left(4/5\right)^2}}+3$
$=1+\frac{1}{2}+\frac{5}{3}+3=\frac{37}{6}$

6. The equation is equivalent to $\left(x+1\right)^5-243=0\Rightarrow x=2$. This is the only real solution because the derivative of the left-hand side is $5\left(x+1\right)^4$, which is always ≥ 0.

7. The sequence of derivatives is $x^{-2},-2x^{-3},6x^{-4},-24x^{-5},120x^{-6},\ldots$, and the general term for this sequence is $\left(-1\right)^{n-1}n!x^{-(n+1)}=\frac{\left(-1\right)^{n-1}n!}{x^{n+1}}$, which isn't equivalent to any of the answer choices.

8. Adding the three equations together gives $13x+13y+13z=169\Rightarrow x+y+z=13$. Therefore, $a+b+c=13$.

9. $\frac{1}{15-13}\int_{13}^{15}H\sqrt{H^2-144}dH=\frac{1}{2}\cdot\frac{1}{3}\left(H^2-144\right)^{1.5}\Big|_{13}^{15}=\frac{1}{6}\left(729-125\right)=\frac{604}{6}\approx 100.7$, so the closest whole number is 101.

10. Let $d = \lim_{n\to\infty} d_n$. Then $d = \frac{d^2al^2 + as}{d} \Rightarrow d^2\left(1 - al^2\right) = as \Rightarrow d = \sqrt{\frac{as}{1-al^2}} = \sqrt{\frac{-as}{al^2 - 1}}$.

11. Using only 1 small triangle, there are 16. Using only 4 small triangles, there are 7. Using only 9 small triangles, there are 3. Using all 16 small triangles, there is 1. 16+7+3+1=27

12. This region consists of two cones, one with radius 7 and height 21, the other with radius 2 and height 6. $\frac{1}{3}\pi\left((7)^2 21 + (2)^2 6\right) = 351\pi$

13. Let $x = \text{the number of Batman riders}$ and $y = \text{the number of Superman riders}$. 20 students didn't ride any ride, so the number of Batman and/or Superman riders was $2000 - 20 - 33 = 1947$. Also, $.24y = .15x \Rightarrow y = \frac{5}{8}x$. Therefore, $1947 = x + y - .15x$ $= x + \frac{5}{8}x - \frac{3}{20}x = \frac{59}{40}x \Rightarrow x = \frac{40}{59}(1947) = 1320$.

14. $\sum_{x=1}^{501} \frac{1}{x^2 + 3x + 2} = \sum_{x=1}^{501}\left(\frac{1}{x+1} - \frac{1}{x+2}\right) = \frac{1}{2} - \frac{1}{503} = \frac{501}{1006}$, so $m - n = 501 - 1006 = -505$

15. $\frac{\left(\sin^2 x + \cos^2 x\right)\left(1 + \cot^2 x\right)\tan x}{\left(1 + \tan^2 x\right)\cot x} = \frac{\csc^2 x \tan x}{\sec^2 x \cot x} = 1$, so the integral equals $1 - 0.5 = 0.5$

16. $1 = C\int_0^\infty x^2 e^{-2x^3}\,dx = -\frac{C}{6}\left(e^{-2x^3}\right)\Big|_0^\infty = \frac{C}{6} \Rightarrow C = 6$

17. The two graphs intersect when $x = -5/3$ and $x = 1$, and the line is above the parabola, so the area is $\int_{-5/3}^{1}\left(5x + 6 - 6x^2 - 9x + 4\right)dx = \int_{-5/3}^{1}\left(10 - 4x - 6x^2\right)dx$ $= \left(10x - 2x^2 - 2x^3\right)\Big|_{-5/3}^{1} = 10 - 2 - 2 + \frac{50}{3} + \frac{50}{9} - \frac{250}{27} = \frac{512}{27}$

18. Multiplying by the conjugate, $\lim_{x\to\infty}\left(\sqrt{2x^2 + 7x} - \sqrt{2x^2 + 3x}\right) = \lim_{x\to\infty}\frac{4x}{\sqrt{2x^2 + 7x} + \sqrt{2x^2 + 3x}}$ $= \frac{4}{2\sqrt{2}} = \sqrt{2}$

19. $v(t)=6t^2+4t+8$, and $v(t)=40$ when $t=2$. Since $v(t)>0$, the distance the car travels is $\int_0^2 \left(6t^2+4t+8\right)dt=\left(2t^3+2t^2+8t\right)\Big|_0^2=16+8+16=40$.

20. $\int_1^2\int_0^{1.5}\frac{x}{\sqrt{9-y^2}}\,dydx=\int_1^2 x\sin^{-1}\frac{y}{3}\Big|_0^{1.5}dx=\frac{\pi}{6}\int_1^2 xdx=\frac{\pi}{6}\left(\frac{1}{2}x^2\right)\Big|_1^2=\frac{\pi}{6}\cdot\frac{3}{2}=\frac{\pi}{4}$

21. $\frac{dy}{dx}=\frac{dy}{d\left(x^{-1}\right)}\cdot\frac{d\left(x^{-1}\right)}{dx}\Rightarrow\frac{dy}{d\left(x^{-1}\right)}=\frac{dy/dx}{d\left(x^{-1}\right)/dx}=\frac{\dfrac{3x^2\cdot\frac{1}{x}-6x\ln x}{9x^4}}{-\frac{1}{x^2}}=\frac{3x\left(1-2\ln x\right)\left(-x^2\right)}{9x^4}$
$=\frac{2\ln x-1}{3x}$

22. $\lim_{x\to0}\frac{\csc^2 x\left(\cos x-1\right)\tan x}{x\sec x}=\lim_{x\to0}\frac{\cos x-1}{x\sin x}=\lim_{x\to0}\frac{-\sin x}{x\left(\cos x+1\right)}=\lim_{x\to0}\frac{-\cos x}{-x\sin x+\cos x+1}$
$=-\frac{1}{2}$

23. $\frac{dL}{dt}=\frac{t+1}{L}\Rightarrow\int LdL=\int(t+1)dt\Rightarrow\frac{1}{2}L^2=\frac{1}{2}t^2+t+C$, and since the train traveled 2 miles in 1 minute, $\frac{1}{2}(2)^2=\frac{1}{2}(1)^2+1+C\Rightarrow C=\frac{1}{2}\Rightarrow L^2=t^2+2t+1\Rightarrow L=t+1$. Since $L=11$ is when the train would be at the town, a total of 10 minutes is all that the officials have, but 1 minute has already elapsed, so they have 9 minutes left.

24. The directrices are a distance of a^2/c units away from the center on either side and run perpendicular to the major axis. Therefore, the area is a rectangle with side lengths of $2a^2/c$ on a side. For the ellipse, $a^2=25$ and $c=4$, so that side length is $\frac{25}{2}$. For the hyperbola, $a^2=9$ and $c=5$, so that side length is $\frac{18}{5}$. $\frac{25}{2}\cdot\frac{18}{5}=45$

25. $P(n)=-\frac{1}{4}n^2+6n-\ln(n+1)-1000\Rightarrow P'(n)=-\frac{1}{2}n+6-\frac{1}{n+1}$. $P'(n)=0$ when $\frac{-n^2-n+12n+12-2}{2(n+1)}=0\Rightarrow n^2-11n-10=0\Rightarrow n=\frac{11+\sqrt{161}}{2}\approx11.8$ (since $n>0$), and this does generate a maximum, so the nearest whole number is 12.

26. $x_2 = 0 - \frac{1}{-4} = \frac{1}{4} \Rightarrow x_3 = \frac{1}{4} - \frac{1/32}{-29/8} = \frac{1}{4} + \frac{1}{116} = \frac{30}{116} = \frac{15}{58}$

27. The derivatives cycle $-\cos x \to \sin x \to \cos x \to -\sin x \to -\cos x$, so the 2011th derivative coincides with $-\sin x$.

28. $\int (6y+2)dy = \int (\sin x + e^x)dx \Rightarrow 3y^2 + 2y = -\cos x + e^x + C$, and since $\left(0, \frac{4}{3}\right)$ is on the graph, $3\left(\frac{4}{3}\right)^2 + 2\left(\frac{4}{3}\right) = -\cos 0 + e^0 + C \Rightarrow C = 8$. Therefore, $3y^2 + 2y = -\cos x + e^x + 8$ $\Rightarrow 8 - \cos x - 2y + e^x - 3y^2 = 0$.

29. $14x^3 y \frac{dy}{dx} + 21x^2 y^2 + \frac{3y - 3x\,dy/dx}{y^2} + 12y^2 \frac{dy}{dx} + 10x = 0 \Rightarrow -112\frac{dy}{dx} + 84 - 3 - 6\frac{dy}{dx}$ $+12\frac{dy}{dx} + 20 = 0 \Rightarrow -106\frac{dy}{dx} = -101 \Rightarrow \frac{dy}{dx} = \frac{101}{106}$

30. $P(2500, F) = 50F^4 - 4000F^3 \Rightarrow P'(2500, F) = 200F^3 - 12000F^2 = 200F^2(F - 60)$, which is positive for positive values $F > 60$.