Answers:

1. B
2. D
3. C
4. C
5. A
6. B
7. E
8. A
9. C
10. C
11. D
12. B
13. A
14. E
15. B
16. B
17. C
18. A
19. A
20. D
21. B
22. C
23. B
24. C
25. B
26. E
27. A
28. D
29. C
30. C

Solutions:

1. $a_3 = \frac{a_7^2}{a_{11}} = \frac{50^2}{250\sqrt[3]{5}} = \frac{10}{\sqrt[3]{5}} = 2\sqrt[3]{25}$

2. $i^{2(-34)-5(-68)} = i^{-68+340} = i^{272} = 1$

3. Josh is 100 feet west and 240 feet south of Katie, so he is a distance of 260 feet from her. Since he can walk 5 feet per second, he must walk for 52 seconds, or $13/15$ minutes.

4. $0 = 2e^{2x} + 3e^x - 14 = (2e^x + 7)(e^x - 2) \Rightarrow x = \ln 2$ only. So the sum is $\ln 2$.

5. The probability of Sarah getting to work and leaving work on time is:
$(0.4)(0.25)(0.5) + (0.6)\left(2/3\right)\left(9/10\right) = 0.05 + 0.36 = 0.41$, or 41%.

6. The sum of the interior angles of a nonagon is $180^\circ(9-2) = 1260^\circ$, and the shortest distance from the point to the line is $\frac{|24(-3) - 7(8) - 497|}{\sqrt{24^2 + 7^2}} = 25$. The greatest common divisor of 1260 and 25 is 5.

7. Mark ate 3 pieces more than Jane, and the number of calories in 3 pieces is
$\frac{3}{8}\pi(3)^2(10) + \frac{3}{8}\pi(4^2 - 3^2)(15) = \frac{585\pi}{8} \approx \frac{585}{8} \cdot \frac{22}{7} \approx 229.8$, so 230 is the answer.

8. $\frac{90 \cdot 50}{90 + 50} = \frac{225}{7}$

9. All of the product equals 1 except $(2+\sqrt{3})(2+\sqrt{3}) = 7 + 4\sqrt{3}$

10. $A^{-1} = \frac{1}{-15}\begin{bmatrix} 3 & -6 \\ -1 & -3 \end{bmatrix} = \begin{bmatrix} -1/5 & 2/5 \\ 1/15 & 1/5 \end{bmatrix}$

11. Miles ran a total of 72 yards in a total of 9 seconds, so he is averaging 8 yds/sec.

12. $\dfrac{3}{(i+2)^4(i-2)^4}=\dfrac{3}{(-5)^4}=\dfrac{3}{625}$

13. Let $2a$ and $2b$ be the lengths of the diagonals. The circle has radius 2.4, so $ab=5(2.4)=12$, and $a^2+b^2=25$. Therefore, $(a+b)^2=a^2+b^2+2ab=25+2(12)=49$ $\Rightarrow a+b=7$ and $ab=12 \Rightarrow a=4$ and $b=3$. Therefore, the longer diagonal has length 8.

14. $x=2+\cfrac{3}{2+\cfrac{3}{2+\cfrac{3}{2+\ldots}}}=2+\dfrac{3}{x}\Rightarrow 0=x^2-2x-3=(x-3)(x+1)\Rightarrow x=3$

$y=2\sqrt{3+2\sqrt{3+2\sqrt{3+\ldots}}}=2\sqrt{3+y}\Rightarrow 0=y^2-4y-12=(y-6)(y+2)\Rightarrow y=6$

$x+y=3+6=9$

15. If Kyle uses 3 quarters, there is 1 way to pay the meter (3Q).
If Kyle uses 2 quarters, there are 3 ways to pay the meter (5N; 3N,1D; 1N,2D).
If Kyle uses 1 quarter, there are 5 ways to pay the meter (8N,1D; 6N,2D; 4N,3D; 2N,4D; 5D).
If Kyle uses 0 quarters, there are 4 ways to pay the meter (7N,4D; 5N,5D; 3N,6D; 1N,7D).
Therefore, there are a total of 1+3+5+4=13 ways to pay the meter.

16. $2-\sqrt[3]{x}=5\Rightarrow x=-27$, so $f(5)=\dfrac{\sqrt[5]{5-(-27)}}{\sqrt[3]{(-27)^2}}=\dfrac{2}{9}$

17. Let $x=$ the number of students who like all 3 teams. Making a Vinn diagram,
$x+(71-x)+(100-x)+(91-x)+(14+x)+(73+x)+(57+x)+12=442$
$x+418=442\Rightarrow x=24$

18. Completing the square, the center of the ellipse is at the point $(2,-3)$, and the slope of the given line is $1/5$, so the slope of the wanted line is -5. Therefore, the line has equation $y+3=-5(x-2)\Rightarrow 5x+y=7$

19. $1_2+12_3+123_4+1234_5=1+5+27+194=227=1015_6$

20. $m\angle r+m\angle s=180^\circ \Rightarrow m\angle s=180^\circ-m\angle r=180^\circ-97^\circ=83^\circ$

21. Navin's first card is an ace if the first person's first card is either an ace or not. If that person's first card is an ace, the probability is $\left(\frac{4}{52}\right)\left(\frac{3}{51}\right)=\frac{1}{221}$. If that person's first card isn't an ace, the probability is $\left(\frac{48}{52}\right)\left(\frac{4}{51}\right)=\frac{16}{221}$. Therefore the probability is $\frac{1}{221}+\frac{16}{221}=\frac{17}{221}=\frac{1}{13}$.

22. $\frac{5\cdot 5!+6\cdot 6!}{41}=\frac{5!(5+36)}{41}=5!=120$

23. Kim paid $\frac{4}{5}(120)\left(\frac{11}{10}\right)=\frac{528}{5}=105.60$. Chloe paid $120-15=105$, so Chloe paid less.

24. $\frac{1}{\left(\sqrt[3]{5}+\sqrt[3]{7}\right)}\cdot\frac{\left(\sqrt[3]{25}-\sqrt[3]{35}+\sqrt[3]{49}\right)}{\left(\sqrt[3]{25}-\sqrt[3]{35}+\sqrt[3]{49}\right)}=\frac{\sqrt[3]{25}-\sqrt[3]{35}+\sqrt[3]{49}}{12}$

25. The volume of the glass is $\pi(4)^2(10)=160\pi$, and the volume of the cubes is $6(2)^3=48$, so the space left for lemonade is $160\pi-48$.

26. The asymptotes have equations $y=\pm\frac{4}{7}x$, which intersect the line $x=28$ at the points $(28,16)$ and $(28,-16)$, and the distance between these points (the base) is 32. The asymptotes intersect at the origin, which is a distance of 28 away from the base, so the area enclosed is $\frac{1}{2}(32)(28)=448$.

27. The perimeter is $\frac{5}{6}$ of the circumference for each circle, or $\frac{5}{2}$ of the circumference of one circle. Since $10\pi=\frac{5}{2}(2\pi r)=5\pi r\Rightarrow r=2$ for each circle, the area enclosed is $\frac{2^2\sqrt{3}}{4}-\left(3\cdot\left(\frac{1}{6}\pi(2)^2-\frac{2^2\sqrt{3}}{4}\right)\right)=4\sqrt{3}-2\pi$.

28. The figure is two cones, one with radius 2 and height 3, the other with radius $4/3$ and height 2. The total volume is $\frac{1}{3}\pi\left(2^2\cdot 3+\left(4/3\right)^2\cdot 2\right)=\frac{140\pi}{27}$.

29. The number of ways to select the officers is ${}_{12}P_3 = 12\cdot 11\cdot 10$, and the number of ways of selecting the directors is $\binom{12}{5} = \frac{12\cdot 11\cdot 10\cdot 9\cdot 8}{5\cdot 4\cdot 3\cdot 2\cdot 1} = 11\cdot 9\cdot 8$. Therefore, the total number of combinations of officers and directors is $12\cdot 11\cdot 10\cdot 11\cdot 9\cdot 8$ $= 2^6\cdot 3^3\cdot 5\cdot 11^2$.

30. Since the difference of the radii lengths is 3 and the common external tangent has length 3, $\angle ASE = 90^\circ + 45^\circ = 135^\circ$, which is $\frac{3}{8}$ of the circumference of circle S. $\frac{3}{8}(2\pi)(5) = \frac{15\pi}{4}$.