Answers:

1. 2
2. $\frac{6\sqrt{57}}{19}$
3. $7/25$
4. $56/65$
5. 2
6. $\frac{\pi\sqrt{2}}{4}$
7. 32:44 (32 minutes, 44 seconds)
8. $5\pi/6$
9. 2.2
10. $\pi/3$
11. $3/10$
12. 2
13. 1
14. 14
15. $-1/2$
16. $\pi/6$
17. $\pi/4$
18. 20
19. $-\sqrt{3}/3$
20. $-8-8\sqrt{3}$
21. $1/3$
22. 5
23. 13
24. $17\sqrt{3}$
25. D

Solutions:

1. $\sin^2 12^\circ + \sin^2 40^\circ + \sin^2 50^\circ + \sin^2 78^\circ = \sin^2 12^\circ + \sin^2 40^\circ + \cos^2 40^\circ + \cos^2 12^\circ = 1+1$
$= 2$

2. Since $\angle C$ is the largest angle, side c is the longest side.
$c^2 = 4^2 + 6^2 - 2(4)(6)\cos 120^\circ = 76 \Rightarrow c = 2\sqrt{19}$. The area enclosed by the triangle is
$\frac{1}{2}(4)(6)\sin 120^\circ = 6\sqrt{3}$, so the altitude to side c is $h = \frac{2(6\sqrt{3})}{2\sqrt{19}} = \frac{6\sqrt{57}}{19}$.

3. $\cos\left(2\cos^{-1}\left(\frac{4}{5}\right)\right) = 2\cos^2\left(\cos^{-1}\left(\frac{4}{5}\right)\right) - 1 = 2\cdot\frac{16}{25} - 1 = \frac{7}{25}$

4. $\left|\cos(x-y)\right| = \left|\cos x \cos y + \sin x \sin y\right| = \left|\left(\frac{12}{13}\right)\left(-\frac{3}{5}\right) + \left(\frac{5}{13}\right)\left(-\frac{4}{5}\right)\right| = \left|-\frac{56}{65}\right| = \frac{56}{65}$

5. This is equivalent to asking how many times $f(x) = 5\cos x + 1$ intersects the x-axis on the interval $[-\pi, \pi]$, one time around the unit circle. Cosine is negative in two quadrants, so there are 2 points of intersection.

6. $\sin x + \cos x = A\sin(x+B) = A\cos B \sin x + A\sin B \cos x \Rightarrow A\cos B = A\sin B = 1 \Rightarrow$
$A^2 = A^2\left(\sin^2 B + \cos^2 B\right) = (A\sin B)^2 + (A\cos B)^2 = 1+1 = 2 \Rightarrow A = \sqrt{2}$ and
$\sin B = \cos B = \frac{\sqrt{2}}{2}$, thus making the smallest value of $B = \frac{\pi}{4}$. Therefore, $AB = \frac{\pi\sqrt{2}}{4}$.

7. $90 = \frac{1}{2}\left|60\cdot 5 - 11m\right| \Rightarrow 300 - 11m = \pm 180 \Rightarrow 11m = 120 \text{ or } 480 \Rightarrow m = 10\frac{10}{11}$ or
$m = 43\frac{7}{11}$, so the difference between these times is $43\frac{7}{11} - 10\frac{10}{11} = 32\frac{8}{11}$ minutes.
$\frac{8}{11}$ of one minute is $\frac{8}{11}\cdot 60 = \frac{480}{11} = 43\frac{8}{11}$ seconds, so to the nearest second, the elapsed time is 32 minutes and 44 seconds (or 32:44).

8. Squaring both equations, then adding them together gives $9\sin^2 A + 9\cos^2 A$
$+16\sin^2 B + 16\cos^2 B + 24(\sin A\cos B + \cos A \sin B) = 37$, making $\sin(A+B) = \frac{1}{2}$ or
$A+B = 30^\circ$ or $A+B = 150^\circ$. Since $\sin B \le \frac{1}{4}$ by the second equation, $B \le 30^\circ$. Since A

and B are both acute, $A+B=30^\circ$ must be true, making $C=150^\circ$, which in radians is $C=\frac{5\pi}{6}$.

9. $d=4$ when $\cos\left(\frac{\pi}{5.4}(t-4)\right)=\frac{1}{2}\Rightarrow\frac{\pi}{5.4}(t-4)=\frac{\pi}{3}+2\pi k$ or $\frac{5\pi}{3}+2\pi k\Rightarrow t-4=$ $1.8+10.8k$ or $9+10.8k\Rightarrow t=5.8+10.8k$ or $13+10.8k$. The smallest positive value of t that works is $t=13+10.8(-1)=2.2$.

10. $2\cos^2 x-5\cos x+2=0\Rightarrow(2\cos x-1)(\cos x-2)=0\Rightarrow\cos x=\frac{1}{2}$. The smallest positive value is $x=\frac{\pi}{3}$.

11. $33=\tan(x+y)=\frac{\tan x+\tan y}{1-\tan x\tan y}=\frac{3+\tan y}{1-3\tan y}\Rightarrow 33-99\tan y=3+\tan y\Rightarrow$ $30=100\tan y\Rightarrow\tan y=\frac{3}{10}$

12. $\frac{\cos 87^\circ}{\sin 1^\circ}-\frac{\sin 87^\circ}{\cos 1^\circ}=\frac{\cos 87^\circ\cos 1^\circ-\sin 87^\circ\sin 1^\circ}{\sin 1^\circ\cos 1^\circ}=\frac{\cos(87+1)^\circ}{\frac{1}{2}\sin(2\cdot 1^\circ)}=\frac{2\cos 88^\circ}{\sin 2^\circ}=2$

13. Since $\tan(90^\circ-x)=\cot x$, $\tan x\tan(90^\circ-x)=1$. Therefore, $\prod_{i=1}^{89}\tan i^\circ=1^{44}\tan 45^\circ=1$

14. $\sin^2 x-\sin x=1-\sin^2 x\Rightarrow 2\sin^2 x-\sin x-1=0\Rightarrow(2\sin x+1)(\sin x-1)=0\Rightarrow$ $\sin x=1$ or $\sin x=-\frac{1}{2}\Rightarrow x=\frac{\pi}{2},\ \frac{7\pi}{6}$, or $\frac{11\pi}{6}$. $\frac{\pi}{2}+\frac{7\pi}{6}+\frac{11\pi}{6}=\frac{7\pi}{2}$, so $ab=14$.

15. Let $x=\cos^{-1}\left(-\frac{1}{2}\right)$. $\cos\left(\frac{4\pi}{3}-x\right)=\cos\frac{4\pi}{3}\cos x+\sin\frac{4\pi}{3}\sin x=\left(-\frac{1}{2}\right)^2+\left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$ $=\frac{1}{4}-\frac{3}{4}=-\frac{1}{2}$

16. $f(x) = -5\sin\left(7x + \frac{\pi}{3}\right) + 4$ has maximum value 9, which occurs when $\sin\left(7x + \frac{\pi}{3}\right) = -1$.

Therefore, $7x + \frac{\pi}{3} = \frac{3\pi}{2} + 2\pi k \Rightarrow 7x = \frac{7\pi}{6} + 2\pi k \Rightarrow x = \frac{\pi}{6} + \frac{2\pi}{7}k$. The smallest positive value is $\frac{\pi}{6}$.

17. $\left|\langle 1,3\rangle\right| = \sqrt{1^2 + 3^2} = \sqrt{10}$ and $\left|\langle -2,4\rangle\right| = \sqrt{(-2)^2 + 4^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$. Therefore,

$1(-2) + 3\cdot 4 = \sqrt{10}\cdot 2\sqrt{5}\cos\theta \Rightarrow \frac{1}{\sqrt{2}} = \cos\theta \Rightarrow \theta = \frac{\pi}{4}$.

18. $\frac{\sin\frac{\pi}{10}}{z} = \frac{\sin\frac{\pi}{20}}{10} \Rightarrow z = \frac{10\sin\frac{\pi}{10}}{\sin\frac{\pi}{20}} = \frac{20\sin\frac{\pi}{20}\cos\frac{\pi}{20}}{\sin\frac{\pi}{20}} = 20\cos\frac{\pi}{20} \Rightarrow A = 20$

19. $\cot\left(-\frac{4\pi}{3}\right) = -\cot\frac{4\pi}{3} = -\cot\frac{\pi}{3} = -\frac{\sqrt{3}}{3}$

20. $\left(2\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)\right)^4 = 2^4\left(\cos\frac{10\pi}{3} + i\sin\frac{10\pi}{3}\right) = 16\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) = -8 - 8\sqrt{3}i \Rightarrow$

$a + b = -8 - 8\sqrt{3}$

21. $x = \cos\theta = \frac{4x+3y}{5} \Rightarrow 5x = 4x + 3y \Rightarrow x = 3y \Rightarrow \tan\theta = \frac{y}{x} = \frac{1}{3}$

22. $\frac{\sin A}{a} = \frac{\sin B}{b} \Rightarrow \frac{1/2}{a} = \frac{\sin B}{10} \Rightarrow a\sin B = 10\cdot\frac{1}{2} = 5$

23. r is largest when $\cos(7\theta) = -1 \Rightarrow r = 7 - 6(-1) = 7 + 6 = 13$

24. $(x-4)^2 + (y-1)^2 = \sqrt{17}^2 + \sqrt{17}^2 - 2\left(\sqrt{17}\right)^2\cos\theta \Rightarrow$

$x^2 + y^2 + 16 + 1 - 8x - 2y = 34 - 34\cos\frac{\pi}{6} \Rightarrow 34 - 8x - 2y = 34 - 34\cdot\frac{\sqrt{3}}{2} \Rightarrow$

$8x + 2y = 17\sqrt{3}$

25. $\sin^2(90^\circ - x) + \sin^2 x = \cos^2 x + \sin^2 x = 1$, so A is true

$\left(\dfrac{1}{\sin^2 x}\right)-\left(\dfrac{1}{\tan x}\right)^2=\csc^2 x-\cot^2 x=1$, so B is true

$\cos^2(-x)+\sin^2(-x)=1$ (true for any angle), so C is true

$\left(\cos^2 x\right)\left(\tan^2 x-1\right)=1$ would be true if the $-$ was a $+$, so D is false