Answers:

1. D
2. B
3. E
4. B
5. C
6. B
7. C
8. C
9. E
10. A
11. C
12. E
13. D
14. B
15. C
16. D
17. B
18. A
19. A
20. C
21. A
22. E
23. C
24. B
25. A
26. B
27. B
28. D
29. C
30. A

Solutions:

1. The denominator is the factorial of the exponent, beginning with 0, so it's D.

2. $\cosh x = \frac{e^x + e^{-x}}{2} = 1 + \frac{x^2}{2} + \frac{x^4}{24} + \dots = \sum_{i=0}^{\infty} \frac{x^{2i}}{(2i)!}$

3. $\sum_{n=1}^{\infty} n\left(\frac{1}{4} - \frac{1}{4}i\right)^{n-1} = \frac{1}{\left(1-\left(\frac{1}{4}-\frac{1}{4}i\right)\right)^2} = \frac{1}{\left(\frac{3}{4}+\frac{1}{4}i\right)^2} = \frac{16}{(3+i)^2} = \frac{16}{8+6i} = \frac{16(8-6i)}{100}$

$= \frac{32}{25} - \frac{24}{25}i$

4. Using partial fraction decomposition, $\frac{5z+1}{z^2+z-2} = \frac{3}{z+2} + \frac{2}{z-1} = \frac{3}{2}\left(\frac{1}{1-\left(-z/2\right)}\right)$

$-2\left(\frac{1}{1-z}\right)$, and the z^2 term is $\frac{3}{2}\left(-\frac{1}{2}\right)^2 - 2 = \frac{3}{8} - 2 = -\frac{13}{8}$

5. The total number of ways to arrange the letters is $11!$, and the total number of ways to get POWER at the front is $1 \cdot 1 \cdot 1 \cdot 3 \cdot 2 \cdot 6!$, so the probability it $\frac{6 \cdot 6!}{11!} = \frac{1}{9240}$.

6. The graph is a hyperbola with center on $x = 0$ with horizontal transverse axis, so revolving it about the vertical line containing the transverse axis would look like a nuclear cooling tower.

7. Since the waterwheel rotates at 6 revolutions/minute, the period is 10 seconds, meaning $B = \frac{2\pi}{10} = \frac{\pi}{5}$. The amplitude is 7 meters since this is the radius of the waterwheel, and the high and low points are 13 and -1, respectively. Therefore, the equation is $d = 6 + 7\cos\left(\frac{\pi}{5}(t-2)\right)$.

8. $\csc x \cos^2 x + \sin x = \frac{\cos^2 x}{\sin x} + \sin x = \frac{\cos^2 x + \sin^2 x}{\sin x} = \frac{1}{\sin x} = \csc x$

9. $\left(1 - 1/3\right)\left(1 - 1/4\right)\left(1 - 1/5\right)\dots\left(1 - 1/n\right) = \frac{2}{3} \cdot \frac{3}{4} \cdot \frac{4}{5} \cdot \dots \cdot \frac{n-1}{n} = \frac{2}{n}$

10. $768 = y^y + 2y\left(y^{y-1}\right) = 3y^y \Rightarrow y^y = 256 \Rightarrow y = 4$

11. $f(0) = f(2\cdot 0) = 2f(0) \Rightarrow f(0) = 0$, so I is false. $2 = f(1) = f(2\cdot 0.5) = 2f(0.5)$ $\Rightarrow f(0.5) = 1 \Rightarrow f(2.5) = f(5\cdot 0.5) = 5f(0.5) = 5$, so II is true. $f(10) = f(4\cdot 2.5)$ $= 4f(2.5) = 20$, so III is true.

12. $\frac{25}{16} = \left(\frac{5}{4}\right)^2 = (\cos x + \sin x)^2 = 1 + 2\sin x\cos x \Rightarrow 2\sin x\cos x = \frac{9}{16}$. Therefore, $(\cos x - \sin x)^2 = 1 - 2\sin x\cos x = 1 - \frac{9}{16} = \frac{7}{16}$, and since $0 < x < \pi/4$, $\cos x > \sin x > 0$, implying that $\cos x - \sin x = \frac{\sqrt{7}}{4}$.

13. For the triangle in D, the two smaller sides add up to the third side. By the Triangle Inequality, this is not possible for a triangle.

14. For this to work, $r = n - k$, so the smallest values that work are $r = 1$, $k = 2$, and $n = 3$, implying the smallest value of $k + r = 3$.

15. $S = \sum_{i=1}^{\infty} \frac{i}{5^i} = \frac{1}{5} + \frac{2}{25} + \frac{3}{125} + \frac{4}{625} + \ldots$. Multiplying this by $\frac{1}{5}$ gives $\frac{1}{5}S = \frac{1}{25} + \frac{2}{125} + \frac{3}{625} + \ldots$. Subtracting the second equation from the first gives $\frac{4}{5}S = \frac{1}{5} + \frac{1}{25} + \frac{1}{125} + \ldots$ $= \frac{1/5}{1 - 1/5} = \frac{1}{4} \Rightarrow S = \frac{5}{16}$.

16. $x - y - z = -1 - z$, which is negative and could be either even or odd. $xy + z$ is going to be an even number plus z, which could be even or odd. If $(x,y,z) = (1,2,3)$, then $x + y - z = 0$. $xz = y^2 - 1 \Rightarrow xz - y = y^2 - y - 1$, and since $y \geq 2$, $y^2 - y - 1 \geq 1$. Therefore, $xz - y$ is going to be positive. Further, either xz is odd and y is even OR xz is even and y is odd; in either case, $xz - y$ is odd.

17. $0 = 2\left(2^{2z}\right) + 2^z 3^z - 3^{2z} = \left(2\cdot 2^z - 3^z\right)\left(2^z + 3^z\right) \Rightarrow 2 = \left(\frac{3}{2}\right)^z \Rightarrow \frac{1}{2} = \left(\frac{2}{3}\right)^z \Rightarrow z = \log_{2/3}\frac{1}{2}$, so $a = 1/2$

18. I is true, regardless of the integers, because $\frac{a}{b}$ or $\frac{c}{d}$ would first reduce. II is true (just cube the root to verify it equals 8). III is false since $(-8)^{1/3} = -2$, which is not imaginary.

19. $\langle 1,-3,-3\rangle \times \langle 1,-1,-2\rangle = \langle 3,-1,2\rangle$, which has a length of $\sqrt{14}$. Setting $z = 0$ in the two planes yields $x = 2$ and $y = 3$, so the solution would be $\vec{r} = \left(2 + \frac{3}{\sqrt{14}}d\right)\vec{i} + \left(3 - \frac{1}{\sqrt{14}}d\right)\vec{j} + \left(\frac{2}{\sqrt{14}}d\right)\vec{k}$.

20. $\left(16cis\left(80^\circ + 360^\circ n\right)\right)^{1/4} = 2cis\left(20^\circ + 90^\circ n\right)$, so the second quadrant angle is 110°, which makes a 70° angle with the x-axis.

21. The solutions are the roots of the polynomial $f(x) = x^3 - x^2 - 22x + 40$ $= (x-2)(x-4)(x+5)$, so the roots, in numerical order, are $-5,\ 2,$ and 4. Therefore, $a^{c/b} = (-5)^{4/2} = (-5)^2 = 25$.

22. $r = \frac{\sin^2\theta}{\cos^3\theta} \Rightarrow r\cos^3\theta = \sin^2\theta \Rightarrow r^3\cos^3\theta = r^2\sin^2\theta \Rightarrow x^3 = y^2$, so when $y = 8$, $x = 4$.

23. $0 = \begin{vmatrix} -2 & 4k & 5 \\ 0 & 3 & 2 \\ k & -3 & -k \end{vmatrix} = 6k + 8k^2 - 15k - 12 = 8k^2 - 9k - 12$. Since the discriminant is positive, both roots are real, and the sum of the roots is $-\frac{-9}{8} = \frac{9}{8}$.

24. $\sqrt{3}\sin 3x = \cos 2x - \cos 4x = 2\sin 3x \sin x \Rightarrow \sin 3x = 0$ or $\sin x = \sqrt{3}/2$, and the solutions to these two equations are $\pi/3$, $2\pi/3$, π, $4\pi/3$, and $5\pi/3$, so there are 5 solutions.

25. The angle is opposite the side of length $\sqrt{2}$ in a triangle with side lengths of 1, $\sqrt{2}$, and $\sqrt{3}$, which is a right triangle. Therefore, the cotangent is $1/\sqrt{2} = \sqrt{2}/2$.

26. I is false because the slope between the first two points is 34 and the slope between the second and third points is 44. II is true because, by using the method of finite differences, the second set of differences are all 90.

27. Since $\begin{vmatrix} 14 & 13 & 4 \\ 13 & 6 & -2 \\ 4 & -2 & 22 \end{vmatrix} = -2230 \neq 0$ and $(13)^2 - 4(7)(3) = 85 > 0$, I is true. II is false since all circles have an eccentricity of 0. $2x^2 + 3y^2 - 4x + 12y = 4 \Rightarrow \frac{(x-1)^2}{9} + \frac{(y+2)^2}{6} = 1$, so the distance between the center and a focus is $\sqrt{9-6} = \sqrt{3}$, so III is false.

28. $f\left(g\left(\cos\left(\frac{x-\pi}{2}\right)\right)\right) = f\left(16 - \cos^2\left(\frac{x-\pi}{2}\right)\right) = \sqrt{1-\cos^2\left(\frac{x-\pi}{2}\right)} = \sqrt{\sin^2\left(\frac{x-\pi}{2}\right)}$
$= \left|\sin\left(\frac{x-\pi}{2}\right)\right|$, so the range is $[0,1]$.

29. $q(x) = \frac{2x^5 - 14x^3}{x^5 + 4x^4 - 7x^3 - 28x^2} = \frac{2x^3(x^2-7)}{x^2(x^2-7)(x+4)}$, so the only vertical asymptote of the graph is $x = -4$. Since the degrees of the numerator and denominator are the same, the horizontal asymptote is $y = \frac{2}{1} = 2$. There are no oblique asymptotes since there is a horizontal one, so $M + A + O = 2 + 1 + 0 = 3$.

30. The constraint lines intersect at the points $(0,3)$, $(5,1)$, $(-3,2)$, and $(0,-1)$, and the constraints define the area enclosed by the four lines. Plugging in those points to P, $P(0,3) = 27$, $P(5,1) = 34$, $P(-3,2) = 3$, and $P(0,-1) = -9$. Therefore, $M - m$ $= 34 - (-9) = 43$.