Answers:

1. B
2. E
3. A
4. B
5. D
6. A
7. E
8. C
9. B
10. D
11. C
12. D
13. B
14. B
15. B
16. A
17. D
18. B
19. D
20. B
21. C
22. B
23. D
24. C
25. B
26. D
27. C
28. B
29. B
30. B

Solutions:

1. By Rolle's Theorem, $0 = f'(x) = \dfrac{(x+2)(2x-9)-(x^2-9x+14)}{(x+2)^2} = \dfrac{x^2+4x-32}{(x+2)^2}$

$= \dfrac{(x+8)(x-4)}{(x+2)^2} \Rightarrow x = 4$ (because it must be the case that $2 < x < 7$)

2. $xf'(x) = x\left(x^2+x+1+\frac{1}{x}\right) = x^3+x^2+x+1$, which has no minimum value (cubic)

3. $\dfrac{dy}{dx} = \dfrac{r\cos\theta + r'\sin\theta}{-r\sin\theta + r'\cos\theta} = \dfrac{\cos 2\theta \cos\theta - 2\sin 2\theta \sin\theta}{-\cos 2\theta \sin\theta - 2\sin 2\theta \cos\theta}$, so $\left.\dfrac{dy}{dx}\right|_{\theta=\pi/6}$

$= \dfrac{\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - 2\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)}{-\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) - 2\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)} = \dfrac{-\sqrt{3}/4}{-7/4} = \dfrac{\sqrt{3}}{7}$

4. $\lim\limits_{x\to 1} \dfrac{\ln x + e^x - 1}{x^3 - 3x + 5} = \dfrac{0+e-1}{1-3+5} = \dfrac{e-1}{3}$

5. Since f is even, $\int_{-m}^{m} f(x)\,dx = 2\int_0^m f(x)\,dx$, so $5\int_{-m}^{m} f(x)\,dx = 10\int_0^m f(x)\,dx = 10(7) = 70$

6. $\dfrac{1}{4}\int_1^5 \left(4x^2+3x+\dfrac{2}{x}\right)dx = \dfrac{1}{4}\left(\dfrac{4}{3}x^3+\dfrac{3}{2}x^2+2\ln|x|\right)\Big|_1^5 = \dfrac{1}{4}\left(\dfrac{500}{3}+\dfrac{75}{2}+2\ln 5-\dfrac{4}{3}-\dfrac{3}{2}\right)$

$= \dfrac{151}{3} + \dfrac{\ln 5}{2}$

7. By Fundamental Theorem of Calculus, $F'(x) = -\dfrac{2}{5x^6+x^3}\cdot 3x^2 = -\dfrac{6}{5x^4+x}$

8. $\int_{\pi/6}^{\pi/3} \tan^2 x\,dx = \int_{\pi/6}^{\pi/3} (\sec^2 x - 1)\,dx = (\tan x - x)\Big|_{\pi/6}^{\pi/3} = \sqrt{3} - \dfrac{\pi}{3} - \dfrac{\sqrt{3}}{3} + \dfrac{\pi}{6} = \dfrac{2\sqrt{3}}{3} - \dfrac{\pi}{6}$

9. I diverges by p-series test. II diverges by integral test. III diverges by limit comparison test with $1/n$. IV converges by integral test.

10. $14t^2+t-3=(7t-3)(2t+1)$, and a sign chart reveals this is positive for t-values in the intervals $\left(-\infty,-\frac{1}{2}\right)\cup\left(\frac{3}{7},\infty\right)$

11. $f'(x)=\dfrac{1}{2\sqrt{1+x}}$, $f''(x)=-\dfrac{1}{4(1+x)^{3/2}}$, and $f'''(x)=\dfrac{3}{8(1+x)^{5/2}}$, so the third-degree Taylor polynomial centered at $a=0$ is $f(0)+f'(0)x+\dfrac{f''(0)}{2}x^2+\dfrac{f'''(0)}{6}x^3$ $=1+\dfrac{x}{2}-\dfrac{x^2}{8}+\dfrac{x^3}{16}$

12. $\int_0^1\sqrt{1+x}\,dx\approx\int_0^1\left(1+\dfrac{x}{2}-\dfrac{x^2}{8}\right)dx=\left(x+\dfrac{x^2}{4}-\dfrac{x^3}{24}\right)\Big|_0^1=1+\dfrac{1}{4}-\dfrac{1}{24}=\dfrac{29}{24}$

13. $C'(x)=t(3x)g'(x)+3t'(3x)g(x)\Rightarrow C'(2)=t(6)g'(2)+3t'(6)g(2)=9(4)+3(3)(5)$ $=81$

14. $\int_2^5\dfrac{x+3}{x+2}dx=\int_2^5\left(1+\dfrac{1}{x+2}\right)dx=\left(x+\ln|x+2|\right)\Big|_2^5=3+\ln 7-\ln 4$

15. $x'=-e^t\sin t+e^t\cos t\Rightarrow x''=-e^t\cos t-e^t\sin t-e^t\sin t+e^t\cos t=-2e^t\sin t$
$y'=e^t\cos t+e^t\sin t\Rightarrow y''=-e^t\sin t+e^t\cos t+e^t\cos t+e^t\sin t=2e^t\cos t$
$\sqrt{(x'')^2+(y'')^2}=\sqrt{4e^{2t}\sin^2 t+4e^{2t}\cos^2 t}=\sqrt{4e^{2t}}=2e^t$

16. The limit is indeterminate of type 0^0, so $\lim\limits_{x\to0^+}\tan x\ln x=\lim\limits_{x\to0^+}\dfrac{\ln x}{\cot x}=\lim\limits_{x\to0^+}\dfrac{1/x}{-\csc^2 x}$ $=\lim\limits_{x\to0^+}\left(-\dfrac{\sin^2 x}{x}\right)=\lim\limits_{x\to0^+}\left(\dfrac{\sin x}{x}\right)(-\sin x)=1\cdot0=0\Rightarrow\lim\limits_{x\to0^+}x^{\tan x}=e^0=1$.

17. $4c=f'(c)=\dfrac{f(2)-f(0)}{2-0}=\dfrac{9-1}{2-0}=4\Rightarrow c=1$ (which satisfies $0<c<2$)

18. $y_0=2\Rightarrow y_1=2+\dfrac{1}{2}\left(\dfrac{10\cdot0}{0+2}\right)=2\Rightarrow y_2=2+\dfrac{1}{2}\left(\dfrac{10\cdot\frac{1}{2}}{\frac{1}{2}+2}\right)=3$

19. continuity $\Rightarrow 7(2)+2(2)^2=-h(2)^3+j\Rightarrow 22=-8h+j$

differentiability $\Rightarrow 7+4(2)=-3h(2)^2 \Rightarrow 15=-12h \Rightarrow h=-1.25$

Since $22=-8h+j$, $22=-8(-1.25)+j \Rightarrow j=12$

20. $y'=3x^{1/2}$, so the length is $L=\int_0^1 \sqrt{1+9x}\,dx=\frac{2}{27}(1+9x)^{3/2}\Big|_0^1=\frac{2}{27}\left(10^{3/2}-1\right)$

21. $\frac{dy}{dx}=\frac{dy/d\theta}{dx/d\theta}=\frac{\alpha\sin\theta}{\alpha(1+\cos\theta)}=\frac{\sin\theta}{1+\cos\theta}=\tan\frac{\theta}{2}$

22. $\lim_{x\to0}\left(\frac{\sin 4x}{x}\right)\cdot\lim_{x\to0}\left(\frac{1-\cos x}{x^2}\right)=\lim_{x\to0}\left(\frac{4\cos 4x}{1}\right)\cdot\lim_{x\to0}\left(\frac{\sin x}{2x}\right)=4\lim_{x\to0}\left(\frac{\cos x}{2}\right)=4\cdot\frac{1}{2}=2$

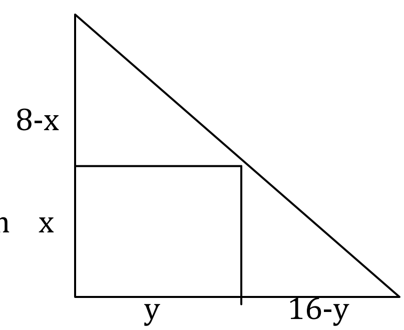
23. $\int_{\pi/4}^{\pi/3}\sec^2 x\tan^2 x\,dx=\frac{1}{3}\tan^3 x\Big|_{\pi/4}^{\pi/3}=\frac{1}{3}\left(3\sqrt{3}-1\right)=\sqrt{3}-\frac{1}{3}$

24. $\lim_{x\to0}\left(\frac{1}{\sin^2 x}-\frac{1}{x^2}\right)=\lim_{x\to0}\left(\frac{x^2-\sin^2 x}{x^2\sin^2 x}\right)=\lim_{x\to0}\left(\frac{2x-\sin 2x}{x^2\sin 2x+2x\sin^2 x}\right)$

$=\lim_{x\to0}\left(\frac{2-2\cos 2x}{2x^2\cos 2x+4x\sin 2x+2\sin^2 x}\right)=\lim_{x\to0}\left(\frac{4\sin 2x}{-4x^2\sin 2x+12x\cos 2x+6\sin 2x}\right)$

$=\lim_{x\to0}\left(\frac{8\cos 2x}{-8x^2\cos 2x-32x\sin 2x+24\cos 2x}\right)=\frac{8}{24}=\frac{1}{3}$

25. By the diagram, $\frac{8-x}{y}=\frac{8}{16}\Rightarrow y=16-2x$, so the area of the rectangular playground is $A=xy=x(16-2x)=16x-2x^2\Rightarrow A'=16-4x$, which is equal to 0 when $x=4$, and this creates a maximum because the graph of A is a parabola opening downward. Therefore, the maximum area is $A=16(4)-2(4)^2=64-32=32$.



26. $x^2\frac{dy}{dx}+2xy+2x=2y\frac{dy}{dx}\Rightarrow\frac{dy}{dx}=\frac{-2xy-2x}{x^2-2y}\Rightarrow\frac{d^2y}{dx^2}$

$$= \frac{\left(x^2-2y\right)\left(-2x\frac{dy}{dx}-2y-2\right)-\left(-2xy-2x\right)\left(2x-2\frac{dy}{dx}\right)}{\left(x^2-2y\right)^2}$$. Also,

$\left.\frac{dy}{dx}\right|_{(1,1)} = \frac{-2\cdot1\cdot1-2\cdot1}{1^2-2\cdot1} = 4$, so

$$\left.\frac{d^2y}{dx^2}\right|_{(1,1)} = \frac{\left(1^2-2\cdot1\right)\left(-2\cdot1\cdot4-2\cdot1-2\right)+\left(2\cdot1\cdot1+2\cdot1\right)\left(2\cdot1-2\cdot4\right)}{\left(1^2-2\cdot1\right)^2} = 12-24 = -12$$

27. $T_4 = \frac{1/2}{2}\left(2+2\left(3.25+5+7.25\right)+10\right) = \frac{43}{4}$, and the exact value of the integral is $\int_1^3\left(x^2+1\right)dx = \left.\left(\frac{1}{3}x^3+x\right)\right|_1^3 = 9+3-\frac{1}{3}-1 = \frac{32}{3}$, so the positive difference in these two numbers is $\left|\frac{43}{4}-\frac{32}{3}\right| = \left|\frac{129-128}{12}\right| = \frac{1}{12}$

28. $f'(x) = -\frac{400}{(x+1)^2}$, so to get $f(n)$ and $f'(n)$ to both be integers when n is a positive integer, $x+1$ must divide 20 and $x+1>1$. Therefore, $x+1$ could be 2, 4, 5, 10, or 20, meaning x must be 1, 3, 4, 9, or 19. $1+3+4+9+19=36$

29. The area of the cross-sections would be $A(x) = \frac{\left(\sqrt{\cos x}\right)^2\sqrt{3}}{4} = \frac{\sqrt{3}}{4}\cos x$. Therefore, the volume would be $\frac{\sqrt{3}}{4}\int_{-\pi/2}^{\pi/2}\cos x\,dx = \frac{\sqrt{3}}{4}\left(\sin x\right)\Big|_{-\pi/2}^{\pi/2} = \frac{\sqrt{3}}{4}\left(1-(-1)\right) = \frac{\sqrt{3}}{2}$.

30. The area would be the sum of 14 rectangles, each of which has a width of 1 and heights of 1, 2, 3, …, 14. Therefore, the area is $1+2+3+\ldots+14 = \frac{14\cdot15}{2} = 105$.