For all questions, answer choice "E) NOTA" means none of the above answers is correct.

1. For what values of x is Rolle's Theorem satisfied on the interval $[2,7]$ for the function $f(x)=\dfrac{x^2-9x+14}{x+2}$?

A) 3 B) 4 C) 5 D) 6 E) NOTA

2. Let $f(x)=\dfrac{x^3}{3}+\dfrac{x^2}{2}+x+\ln|x|$. At what value of x does $xf'(x)$ achieve its minimum value?

A) -3 B) -2 C) -1 D) 0 E) NOTA

3. Find the slope of the tangent line to the curve $r=\cos 2\theta$ when $\theta=\pi/6$.

A) $\sqrt{3}/7$ B) $\sqrt{3}/3$ C) 0 D) $-\sqrt{3}$ E) NOTA

4. Evaluate: $\lim\limits_{x\to 1}\dfrac{\ln x+e^x-1}{x^3-3x+5}$

A) $-1/3$ B) $\dfrac{e-1}{3}$ C) $\dfrac{e+1}{3}$ D) $1/3$ E) NOTA

5. If f is a continuous, even function with $\int_0^m f(x)dx=7$, then $5\int_{-m}^{m} f(x)dx=?$

A) 7 B) 14 C) 35 D) 70 E) NOTA

6. Find the average value of the function $f(x)=\dfrac{4x^3+3x^2+2}{x}$ over the interval $[1,5]$.

A) $\dfrac{151}{3}+\dfrac{\ln 5}{2}$ B) $\dfrac{605}{12}+\dfrac{\ln 5}{2}$ C) $\dfrac{151}{3}+2\ln 5$ D) $\dfrac{605}{12}+2\ln 5$ E) NOTA

7. If $F(x)=\int_{x^3}^{1}\dfrac{2}{5t^2+t}dt$, find $F'(x)$.

A) $\dfrac{2}{5x^6+x^3}$ B) $\dfrac{3}{5x^4+x}$ C) $\dfrac{6}{5x^4+x}$ D) $\ln\left(5x^6+x^3\right)$ E) NOTA

8. Evaluate: $\int_{\pi/6}^{\pi/3} \tan^2 x \, dx$

A) $\frac{2\sqrt{3}}{3}-\frac{\pi}{3}$ B) $\frac{\sqrt{3}}{3}-\frac{\pi}{3}$ C) $\frac{2\sqrt{3}}{3}-\frac{\pi}{6}$ D) $\frac{\sqrt{3}}{3}-\frac{\pi}{6}$ E) NOTA

9. How many of the following series converge?

I) $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}$ II) $\sum_{n=1}^{\infty}\frac{1}{2n+1}$ III) $\sum_{n=1}^{\infty}\frac{n}{n^2+1}$ IV) $\sum_{n=1}^{\infty}\frac{1}{n^2+1}$

A) 0 B) 1 C) 3 D) 4 E) NOTA

10. A function's derivative is given by $h(t)=14t^2+t-3$. For what value(s) of t is the original function strictly increasing as t increases?

A) $\left(-\infty,-1/2\right)$ B) $\left(-1/2,3/7\right)$ C) $\left(-\infty,-1/2\right)\cup\left(-3/7,\infty\right)$ D) $\left(-\infty,-1/2\right)\cup\left(3/7,\infty\right)$
E) NOTA

11. What is the third-degree Taylor polynomial of $f(x)=\sqrt{1+x}$ centered at $a=0$?

A) $1-\frac{x}{2}+\frac{x^2}{8}-\frac{x^3}{16}$ B) $1+\frac{x}{2}+\frac{x^2}{8}+\frac{x^3}{48}$ C) $1+\frac{x}{2}-\frac{x^2}{8}+\frac{x^3}{16}$ D) $1+\frac{x}{4}-\frac{x^2}{24}+\frac{x^3}{32}$ E) NOTA

12. Using only the first three terms of the answer to the previous problem, approximate the value of $\int_0^1 \sqrt{1+x}dx$.

A) $69/96$ B) $5/4$ C) $235/192$ D) $29/24$ E) NOTA

13. Given that t and g are differentiable functions with $t(3)=7$, $t'(3)=2$, $t(6)=9$, $t'(6)=3$, $g(2)=5$, and $g'(2)=4$, find the value of $C'(2)$ if $C(x)=t(3x)g(x)$.

A) 45 B) 81 C) 58 D) 51 E) NOTA

14. Evaluate: $\int_2^5 \frac{x+3}{x+2}dx$

A) $3+\ln 4-\ln 7$ B) $3-\ln 4+\ln 7$ C) $-3+\ln 7-\ln 4$ D) $-3-\ln 7+\ln 4$ E) NOTA

15. The motion of a particle in a plane is given by the parametric equations $x = e^t \cos t$ and $y = e^t \sin t$. What is the magnitude of its acceleration vector at any time t?

A) $2e^t\sqrt{\cos 2t}$ B) $2e^t$ C) e^t D) $2e^{2t}$ E) NOTA

16. Evaluate: $\lim_{x\to 0^+} x^{\tan x}$

A) 1 B) 0 C) -1 D) does not exist E) NOTA

17. Find the value of c guaranteed by the Mean Value Theorem for the function $f(x) = 2x^2 + 1$ over the interval $[0,2]$.

A) 2 B) 3 C) 0 D) 1 E) NOTA

18. Suppose $\frac{dy}{dx} = \frac{10x}{x+y}$, and that $y = 2$ when $x = 0$. Use Euler's Method with two steps of equal size to estimate the value of y when $x = 1$.

A) 1 B) 3 C) 5 D) 7 E) NOTA

19. Let $L(n) = \begin{cases} 7n + 2n^2, & n \le 2 \\ -hn^3 + j, & n > 2 \end{cases}$. For what ordered pair (h, j) will L be continuous and differentiable over all real numbers?

A) $(7,8)$ B) $(11,0)$ C) $(13,-4)$ D) $(-1.25,12)$ E) NOTA

20. Find the length of the curve $y = 2x^{3/2}$ between the points where $x = 0$ and $x = 1$.

A) $\frac{2}{27}\left(10^{3/2}\right)$ B) $\frac{2}{27}\left(10^{3/2} - 1\right)$ C) $\frac{2}{3}\left(10^{3/2}\right)$ D) $\frac{4}{5}$ E) NOTA

21. Let $x = \alpha(\theta + \sin\theta)$ and $y = \alpha(1 - \cos\theta)$, where α is a constant. Find $\frac{dy}{dx}$.

A) $\frac{\cos\theta}{1+\sin\theta}$ B) $\cos\theta + \cot\theta$ C) $\tan\frac{\theta}{2}$ D) $\sin\theta + \tan\theta$ E) NOTA

22. Evaluate: $\lim_{x\to 0}\dfrac{\sin 4x-\sin 4x\cos x}{x^3}$

A) 1 B) 2 C) 4 D) does not exist E) NOTA

23. Evaluate: $\int_{\pi/4}^{\pi/3}\sec^2 x\tan^2 x\,dx$

A) 5 B) $\sqrt{3}-1$ C) $\sqrt{3}$ D) $\sqrt{3}-\dfrac{1}{3}$ E) NOTA

24. Evaluate: $\lim_{x\to 0}\left(\dfrac{1}{\sin^2 x}-\dfrac{1}{x^2}\right)$

A) 0 B) $1/60$ C) $1/3$ D) $1/2$ E) NOTA

25. Bryan needs a rectangular fence to build around a playground for clowns. What is the maximum area for this playground, in square meters, if it is to fit into a right triangular plot with legs measuring 8 meters and 16 meters? Assume two of the sides of the fence lie on the legs of the larger triangular plot.

A) 16 B) 32 C) 48 D) 64 E) NOTA

26. Given $x^2y+x^2=y^2+1$, evaluate $\left.\dfrac{d^2y}{dx^2}\right|_{(x,y)=(1,1)}$.

A) 36 B) -36 C) 12 D) -12 E) NOTA

27. If the definite integral $\int_1^3\left(x^2+1\right)dx$ is approximated using the Trapezoidal Rule with $n=4$, what is the positive difference in the exact value and the approximation?

A) 0 B) $7/3$ C) $1/12$ D) $65/6$ E) NOTA

28. If $f(x)=\dfrac{400}{x+1}$, find the sum of all positive integers n such that $f(n)$ and $f'(n)$ are both integers.

A) 49 B) 36 C) 25 D) 16 E) NOTA

29. Find the volume of the solid whose base is the region bounded above by $y = \sqrt{\cos x}$ and below by the x-axis on the interval $\left[-\pi/2, \pi/2\right]$ and whose cross-sections perpendicular to the x-axis are equilateral triangles.

A) $\sqrt{3}/4$ B) $\sqrt{3}/2$ C) $\sqrt{3}$ D) $3\sqrt{3}/4$ E) NOTA

30. Evaluate $\int_0^{15} \lfloor x \rfloor dx$, where $\lfloor x \rfloor$ represents the greatest integer function.

A) 90 B) 105 C) 120 D) 150 E) NOTA