

Answers:

1. C
2. D
3. A
4. B
5. E
6. C
7. C
8. B
9. C
10. C
11. B
12. A
13. D
14. B
15. B
16. D
17. E
18. A
19. B
20. D
21. C
22. A
23. A
24. C
25. D
26. E
27. D
28. C
29. D
30. A

Solutions:

1. The given line has slope $\frac{3}{2}$, so the perpendicular line has slope $-\frac{2}{3}$, and since it goes through the point $(-2, 3)$, the line has equation $y - 3 = -\frac{2}{3}(x + 2)$
 $\Rightarrow 3(y - 3) = -2(x + 2) \Rightarrow 2x + 3y = 5$.

$$2. \begin{array}{l} \left| \begin{array}{ccc|c} -1 & 4 & & \\ 8 & 2 & 5 & -5 \\ 20 & 4 & -3 & -6 \\ 3 & -1 & 4 & 16 \\ 31 & & & 5 \end{array} \right| \Rightarrow A = \frac{1}{2} |31 - 5| = 13 \end{array}$$

3. $f(2) = 3 \Rightarrow 4a - 2b + c = 3$, and $x = -3$ is a zero $\Rightarrow 9a + 3b + c = 0$. Subtracting the first equation from the second equation gives $5a + 5b = -3 \Rightarrow a + b = -\frac{3}{5}$.

4. Setting the two functions equal gives $|x - 1| = 2x + 2 \Rightarrow x^2 - 2x + 1 = 4x^2 + 8x + 4 \Rightarrow 0 = 3x^2 + 10x + 3 = (3x + 1)(x + 3) \Rightarrow x = -\frac{1}{3}$ or $x = -3$. The only value that makes $2x + 2 > 0$, though, is $x = -\frac{1}{3}$.

5. $g(3) - f(1) = 2f(3) - 3f(2) - f(1) = 2 \cdot 5 - 3 \cdot 1 - (-1) = 8$

6. $3^{4-x} = 9(3^{x^2-2x}) = 3^{x^2-2x+2} \Rightarrow 4 - x = x^2 - 2x + 2 \Rightarrow 0 = x^2 - x - 2 = (x - 2)(x + 1)$, so both roots are real and their sum is 1. $2^{3y-1} = 4^{y^2} = 2^{2y^2} \Rightarrow 3y - 1 = 2y^2 \Rightarrow 0 = 2y^2 - 3y + 1 = (2y - 1)(y - 1)$, so both roots are real and their sum is $\frac{3}{2}$. $1 \cdot \left(\frac{3}{2}\right) = \frac{3}{2}$

7. $x = \sqrt{3 + \sqrt{3 + \sqrt{3 + \dots}}} \Rightarrow x = \sqrt{3 + x} \Rightarrow x^2 - x - 3 = 0 \Rightarrow x = \frac{1 + \sqrt{13}}{2}$ (since $x > 0$).

$$y = \sqrt{2 + \sqrt{2 + \sqrt{2 + \dots}}} \Rightarrow y = \sqrt{2 + y} \Rightarrow y^2 - y - 2 = 0 \Rightarrow y = 2 \text{ (since } y > 0\text{)}. \quad x - y = \frac{1 + \sqrt{13}}{2} - 2 = \frac{-3 + \sqrt{13}}{2}, \text{ so } a + b - c = -3 + 13 - 2 = 8.$$

8. There are 8 possible sequences of three coin tosses, and there are 3 possible tosses of 2 heads and 1 tail (the tail could be any of the three tosses). The odds, then, are 3:5.

$$9. \quad x = \frac{f^{-1}(x) - 1}{f^{-1}(x) + 2} \Rightarrow xf^{-1}(x) + 2x = f^{-1}(x) - 1 \Rightarrow 2x + 1 = f^{-1}(x)(1 - x) \Rightarrow f^{-1}(x) = \frac{2x + 1}{1 - x}$$

$$10. \quad f(x) = 2x^2 - 5x - 8 = 2\left(x - \frac{5}{4}\right)^2 - \frac{89}{8}, \text{ so the minimum value is } -\frac{89}{8} = -11.125$$

$$11. \quad c = \log_a \frac{3}{b} \Rightarrow a^c = \frac{3}{b} \Rightarrow b = \frac{3}{a^c}$$

$$12. \quad \text{Since } f^{-1}(2) = 3, a = f(3) = 2. \text{ Also, since } f(1) = 3, b = f^{-1}(3) = 1. \quad a + b = 2 + 1 = 3$$

$$13. \quad 4! = 24$$

$$14. \quad \sum_{c=2}^7 \frac{1}{c+1} - \sum_{c=2}^7 \frac{1}{c-1} = \frac{1}{7} + \frac{1}{8} - \frac{1}{2} - 1 = \frac{8+7-28-56}{56} = -\frac{69}{56}$$

$$15. \quad AB = (A + 2.5)\left(B - \frac{2}{3}\right) \Rightarrow 0 = 2.5B - \frac{2}{3}A - \frac{5}{3} \text{ and } AB = (A - 2.5)\left(B + \frac{4}{3}\right) \Rightarrow \\ 0 = -2.5B + \frac{4}{3}A - \frac{10}{3}. \text{ Adding these two equations together gives } 0 = \frac{2}{3}A - 5 \Rightarrow \\ A = \frac{15}{2}. \text{ Plugging that back in gives } B = \frac{8}{3}, \text{ so } AB = \frac{15}{2} \cdot \frac{8}{3} = 20.$$

$$16. \quad \text{To have the appropriate sum and product of roots, Joe's equation would be} \\ 0 = 2x^2 - 3x - 2 = (2x + 1)(x - 2), \text{ so the roots are } 2 \text{ and } -\frac{1}{2}.$$

$$17. \quad \sqrt[3]{27 - 54 \cdot 4 + 36 \cdot 4^2 - 8 \cdot 4^3} = \sqrt[3]{-125} = -5$$

$$18. \quad \text{Let } f(x) = ax^2 + bx + c. \quad c = f(0) = 4, \quad 5 = f(1) = a + b + 4 \Rightarrow a + b = 1, \text{ and } 9 = f(-1) \\ = a - b + 4 \Rightarrow a - b = 5. \text{ Therefore, } a = 3 \text{ and } b = -2, \text{ making } f(x) = 3x^2 - 2x + 4 \Rightarrow \\ f(5) = 3 \cdot 5^2 - 2 \cdot 5 + 4 = 75 - 10 + 4 = 69.$$

$$19. \quad 9x^2 + 9y^2 - 54x + 36y = -101 \Rightarrow 9(x - 3)^2 + 9(y + 2)^2 = 16 \Rightarrow (x - 3)^2 + (y + 2)^2 = \frac{16}{9}, \\ \text{so } A = \frac{4}{3}. \quad 25x^2 + 9y^2 + 100x - 90y = -100 \Rightarrow 25(x + 2)^2 + 9(y - 5)^2 = 225 \Rightarrow \\ \frac{(x + 2)^2}{9} + \frac{(y - 5)^2}{25} = 1, \text{ so } B = 2 \cdot 5 = 10. \quad 3A - 2B = 3\left(\frac{4}{3}\right) - 2(10) = 4 - 20 = -16$$

20. Graphing the system, there are 4 points with $y=1$: $x=1, 2, 3, 4$; 4 points with $y=2$: $x=1, 2, 3, 4$; 3 points with $y=3$: $x=1, 2, 3$; 2 points with $y=4$: $x=1, 2$; and 1 points with $y=5$: $x=1$. Therefore, there are 14 points (this could also be done with Pick's Theorem).

$$21. \quad 0 = z\bar{z} + (-3+4i)\bar{z} + (-3-4i)z = (x^2 + y^2) - 3x + 3yi + 4xi + 4y - 3x - 3yi - 4xi + 4y \\ = x^2 - 6x + 9 + y^2 + 8y + 16 - 25 \Rightarrow (x-3)^2 + (y+4)^2 = 25, \text{ so the center is } (3, -4).$$

22. The vertex of this parabola is $(\frac{2}{3}, -\frac{13}{3})$, so over the domain $x \geq 3$, the graph is increasing. When $x=3$, $y=12$, so the range of the function, which is the domain of the inverse of the function, is $y \geq 12$, or $[12, \infty)$

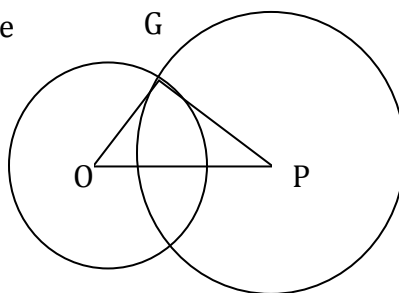
$$23. \quad \frac{3i}{i+3} - \frac{i}{i-3} = \frac{3i(3-i)}{10} - \frac{i(-3-i)}{10} = \frac{2+12i}{10} = \frac{1}{5} + \frac{6}{5}i, \text{ so } a+b = \frac{1}{5} - \frac{6}{5} = -1$$

$$24. \quad \frac{2}{3} = \log_8(x+1) - \log_8(x-1) = \log_8 \frac{x+1}{x-1} \Rightarrow 4 = \frac{x+1}{x-1} \Rightarrow x = \frac{5}{3}, \text{ so } a^b = 5^3 = 125.$$

$$25. \quad \begin{bmatrix} 2 & 5 \\ -2 & -4 \end{bmatrix}^{-1} = \frac{1}{2} \begin{bmatrix} -4 & -5 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} -2 & -2.5 \\ 1 & 1 \end{bmatrix}, \text{ so } AB^{-1} = \begin{bmatrix} -1 & 3 \\ 2 & 5 \\ 0 & -4 \end{bmatrix} \begin{bmatrix} -2 & -2.5 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 5.5 \\ 1 & 0 \\ -4 & -4 \end{bmatrix}.$$

$$26. \quad 0 \geq \frac{x-2}{x-3} - 2 = \frac{-x+4}{x-3} \Rightarrow x < 3 \text{ or } x \geq 4, \text{ or in set } n$$

27. Using Hero's formula, the area of $\triangle GOP$ is $\sqrt{9 \cdot 4 \cdot 3 \cdot 2} = 6\sqrt{6}$, so the length of the altitude to side OP is $\frac{2 \cdot 6\sqrt{6}}{7} = \frac{12\sqrt{6}}{7}$. Therefore, the length of GH is double that altitude, or $\frac{24\sqrt{6}}{7}$.



$$28. \quad 7x^2 - 74 = \begin{vmatrix} -2 & 4 & 0 \\ 3 & x & 3 \\ -1 & 1 & 5 \end{vmatrix} = -10x - 12 + 0 - 0 + 6 - 60 \Rightarrow 0 = 7x^2 + 10x - 8 = (7x-4)(x+2),$$

so the largest value of x that works is $4/7$.

29. Let x_1 and x_2 be the roots of the equation. Since $x_i^2 = 7x_i - 5$, $x_1^2 + x_2^2 = 7(x_1 + x_2) - 10 = 7 \cdot 7 - 10 = 39$. If the smaller acute angle is 39° , the larger acute angle is 51° .
30. $3x^2 + 16x + k = -2x^2 - 4x + 6 \Rightarrow 0 = 5x^2 + 20x + (k - 6)$, which has only one x -value when $20^2 - 4(5)(k - 6) = 0 \Rightarrow k - 6 = 20 \Rightarrow k = 26$.