This portion of the Interschool Test consists of 12 different sections to test your skills in areas as diverse as math, logic, trivia, television, and pop culture. NO SPONSORS ARE TO WORK ON THIS TEST AT ALL—IT SHOULD BE WORKED ONLY BY STUDENTS, AND ONLY THOSE COMPETING AT THE CONVENTION. FAILURE TO ADHERE TO THIS WILL RESULT IN DISQUALIFICATION ON THE INTERSCHOOL TEST. Scoring for each section will be defined at the beginning of each section. Please put all answers on the answer sheet you received at registration. This portion of the Interschool Test will be due by 7:45 pm on Tuesday, 7/19.

Section I – Name That Number

For this section, each item gives a math clue that describes a unique integer. Identify that integer. 4 points each, except #10, which is 14 points.

1. the base-10 representation of the smallest positive three-digit repdigit in base 15
2. the only prime number that is the sum of four consecutive primes
3. the smallest prime number that is not also a Chen prime (a Chen prime is a prime number p such that $p+2$ is either prime or the product of two primes)
4. the base-10 representation of the smallest positive integer that is a palindrome in bases 5 and 9
5. the smallest balanced prime (prime that is the average of its prime predecessor and prime successor), Sophie Germain prime (prime p such that $2p+1$ is also prime), and sum of three consecutive primes
6. the smallest number n such that in a room of n people, the probability that at least two people have the same birthday is larger than 50% (exclude February 29)
7. the largest number n less than 200 such that a pizza can be cut into n pieces, not necessarily identical, using only straight-line cuts
8. the base-10 representation of the smallest positive integer that ends in a 0 when written in bases 2, 3, 5, and 7
9. the base-10 representation of the smallest positive integer that is a three-letter palindrome when written in Roman numerals whose letters are not all the same (e.g., CCC would not work but CXC would)
10. the first number in the sequence of integers of the form $\underbrace{333......3}_{n\text{ 3s}}1$ that is composite

Section II – Movie Quotes with Numbers

For this section, a movie quote that has numbers in it is given. Identify each number in the quote (the length of the blank does not necessarily indicate the length of the number). To make these more difficult, the name of the movie is not given. A one-point bonus will be given if you give the correct name of the movie as well. 5 points each plus bonus.

1. "I know we've only known each other for __ weeks and __ days, but to me, it seems like __

weeks and __ days. The first day seemed like a week, and the second day seemed like __ days, and the third day seemed like a week again, and the fourth day seemed like __ days, but the fifth day, you went to see your mother and that seemed just like a day, but then you came back, and later, on the sixth day in the evening when we saw each other, that started seeming like __ days, so in the evening it seemed like __ days spilling over into the next day, and that started seeming like __ days, so at the end of the sixth day, on into the seventh day, it seemed like a total of __ days. And the sixth day seemed like a week and a half. I have it written down, but I can show it to you tomorrow if you want to see it."

2. "__ days, __ hours, __ minutes, __ seconds. That is when the world will end."

3. "Norman invasion of England."
   "__."
   "That is correct. Magna Carta."
   "__."
   "Yes. Spanish armada."
   "__. __. __. __. __. __. Please do not do that. Come on, I swear…just hang in there one second."

4. "I'm sure that in __ plutonium is available in every corner drug store, but in __ it's a little hard to come by."

5. "Excuse me, gentlemen, this is a $__-a-plate dinner. Good night."
   "Oh, __, oh OK, alright, no problem, here, put us down for, uh, put us down for __."
   "In case we want seconds."

6. "Fast ship? You've never heard of the Millennium Falcon?"
   "Should I have?"
   "It's the ship that made the Kessel Run in less than __ parsecs."

7. "First shalt thou take out the Holy Pin. Then, shalt thou count to __. No more, no less. __ shall be the number thou shalt count, and the number of the counting shall be __. __ shalt thou not count, neither count thou __, excepting that thou then proceed to __. __ is right out."

8. "__ strands of lights, __ individual bulbs per strand, for a grand total of __ imported Italian twinkle lights. __!"

9. "Hey mister. Change? You got change?"
   "Oh, um, sure."
   "__, __, __ dollar. Thanks mister."

10. "__:__. Personal note. When I was a little kid, my mother told me not to stare into the sun. So once when I was __ I did." (this is quoted three different times in the movie; please give all three instances in the order they appear in the movie)

Section III – Somewhat Humorous Proofs

Prove each of the following statements, somewhat humorously. To help you out, for each of the following, use a proof by contradiction for all but question 4. 10 points each.

1. All positive integers greater than 2 are interesting.
2. There exists a rule with no exception.
3. If each day is Opposite Day (where everything is the opposite of what is said) or Not Opposite Day (where everything is as it is said), then you can never tell someone, "Today is Opposite Day," and be accurate.
4. The statement 'The statement "The statement 'The statement "This statement is false." is false.' is false." is false.' is false.
5. Is it impossible to practice moderation in all things.

Section IV – Million-Tap Challenge

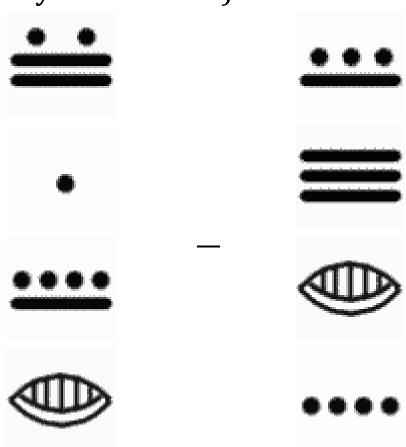
No doubt you have played Million-Tap Challenge on the iPhone in the past. Some of you may have even beat the game. If you have never played, you basically have to tap the screen one million times in order to win. William Stocks beat Million-Tap Challenge in 8 days. Let's say you played similar-style games at the same pace as William Stocks. If you started playing each of the following games tomorrow (July 18, 2011), and you played at the same pace (one million taps every 8 days), on what date (month, day, year) would you theoretically finish each of the following games? Make sure to account for leap years (an extra day, February 29, on every year divisible by 4, except those years divisible by 100 that are not also divisible by 400), and assume the calendar doesn't change in the future. Question 1 is worth 2 points for the correct date; for each of the other three questions, scoring is 2 point for each correct month, 4 points for each correct day, and 10 points for each correct year.

1. Thousand-Tap Challenge
2. Billion-Tap Challenge
3. Trillion-Tap Challenge
4. Quadrillion-Tap Challenge

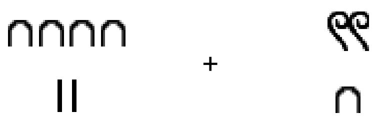
Section V – Math in Different Cultures

For this section, an arithmetic problem using one of the four basic operations is given. However, the numbers are given in a different culture's number system. You must give your answer in the same number system in which the numbers are given for each problem. 10 points each.

1. CDLXIV $\times$ MDCXXIX (Roman numerals, using parentheses notation)
2. (Mayan numerals)



3. $_{,}\varepsilon\upsilon\pi\alpha' \div \kappa\theta'$ (Greek numerals)
4. (Egyptian numerals)



Section VI – Math from A to Z

Identify each word with an association to math that begins with the specified letter. 2 points each.

1. A: A group that commutes
2. B: The last name of two brothers who developed many ideas of calculus and probability
3. C: A nonempty, compact, connected, metric space
4. D: Adjective describing a subset of a space whose closure equals the entire space
5. E: Swiss mathematician known for many results in graph theory
6. F: An object displaying self-similarity on all scales
7. G: French mathematician known for a theory connecting field theory to group theory
8. H: Another name for a cube
9. I: Adjective describing a matrix whose product with itself is the original matrix
10. J: Matrix consisting of all first-order partial derivatives of a vector-valued function
11. K: A quadrilateral with two pairs of adjacent sides that are congruent
12. L: A minor theorem used as a stepping stone to prove a more important theorem
13. M: A function defining a distance between elements of a set
14. N: A function defining a length or size to a vector
15. O: The point at which the three altitudes of a triangle intersect

16. P: The probability a statistical test will reject a false null hypothesis
17. Q: A polygon with four sides
18. R: British mathematician known for a famous paradox in set theory
19. S: Any irrational number that can be expressed as a radical
20. T: Adjective describing a solution that is very simple and of little interest
21. U: Adjective describing a set with more elements than there are positive integers
22. V: The trigonometric function which when added to the cosine equals 1
23. W: Adjective describing a set such that every nonempty subset has a least element
24. X: Logical operator resulting in value of 'True' if exactly one of the operands is 'True'
25. Y: British mathematician who developed a correction for the chi-square test
26. Z: German mathematician known for axioms of set theory in current use

Section VII – Name That Number, Non-Math Edition

For each question, identify the appropriate number(s). 5 points each.

1. the only calendar year in which four different emperors ruled Rome
2. the atomic number of the element in the title of one of Man Or Astro-Man?'s albums
3. the album title that is a number for the singer whose backing band is the Cardinals
4. the title of the 80s anti-Vietnam War song by Paul Hardcastle
5. the total number of miles The Proclaimers would walk before falling down at your door
6. the number at which I can reach Jenny (her name and number were on the wall)
7. the SAT scores of all six main characters on Saved by the Bell
8. the name that Max's computer Euclid gives as the name of God in the movie *Pi*
9. the number in the official name of the largest known dwarf planet in the solar system
10. OK, so this one is math-related. The first instance in the decimal expansion of π where three consecutive digits (e.g., 2, 3, 4) appear in ascending consecutive order (e.g., …234…) occurs in which three places after the decimal? Also, what are the three digits? (Hint: they aren't the digits 2, 3, 4)

Section VIII – Relay

Each question is a math problem, but starting with the second problem, you must use the answer from the previous problem in some way in that problem. For example, the answer to the sixth question is used somehow in the seventh question. The answer to each question is an integer. Each question is worth its number of points (e.g., the 7th question in the relay is worth 7 points). Also, in each question, "TAFTPQ" stands for "the answer from the previous question", so if question 5 references TAFTPQ, that is the answer from question 4. You will have an entire test like this Monday, 7/18, at 8:00 pm.

1. Find the number of degrees in the sum of the exterior angles in a convex 2011-gon.

2. Let $T = \text{TAFTPQ}$. Find the smallest positive integer with exactly $T/8$ positive integral divisors.

3. Let $T = \text{TAFTPQ}$. A rectangular prism with volume T has length 16 and height 15. What is the prism's width?

4. Let $T = \text{TAFTPQ}$. Find the value of $\left(e^{\int_1^T \frac{x}{x^2+1}\,dx}\right)^2$.

5. Let $T = \text{TAFTPQ}$. Two prime numbers A and B, with $A < B$, differ by exactly $T+1$. Find the value of A such that $A+B$ is as small as possible.

6. Let $T = \text{TAFTPQ}$. A function $t : \mathbb{Z}\times\mathbb{Z}\times\mathbb{Z}\rightarrow\mathbb{Z}$ is defined by $t(x,y,z)=\begin{cases} y, \text{ if } x \le y \\ t\big(t(x-1,y,z),t(y-1,z,x),t(z-1,x,y)\big), \text{ otherwise} \end{cases}$. Find the value of $t(T,T-2,T+3)$.

7. Let $T = \text{TAFTPQ}$. If x and y are integers and $x+(T-7)y$ is divisible by 13, then $Ax+y$ is also divisible by 13. Find the sum of all positive integer values of A such that $A<100$.

8. Let $T = \text{TAFTPQ}$. Find the slope of the tangent to the graph of $y = 0.875x^2 - 415x + 127$ at the point where $x = T$.

9. Let $T = \text{TAFTPQ}$. Find the smallest integer value of n such that a convex n-gon has at least T total diagonals.

10. Let $T = \text{TAFTPQ}$. Find the sum of all three-digit positive integers x such that x is equal to T times the sum of the digits of x.

Section IX – Sequences

For this section, determine the designated term(s) in the sequence given. Unless otherwise specified, all sequences begin with the term with subscript 1. 5 points each.

1. the first number that appears twice in the sequence $a_{n+1} = n +$ the sum of the squares of the digits of a_n for $n \ge 1$, with $a_1 = 7$

2. the 2011th term in the sequence $a_{n+1} = \dfrac{a_n}{a_{n-1}}$ for $n \ge 2$, with $a_1 = 1$ and $a_2 = 2$

3. the 25th term in the sequence $a_n = \dfrac{\left(\dfrac{1+\sqrt{5}}{2}\right)^n - \left(\dfrac{1-\sqrt{5}}{2}\right)^n}{\sqrt{5}}$

4. the 2011th term in the sequence $a_{n+1} =$ the sum of all bases and exponents in the prime factorization of a_n for $n \geq 1$, with $a_1 = 219$ (NOTE: any prime p which appears only once in a prime factorization should be written as p^1 for the purposes of this problem)

5. the 2011th term in the sequence $a_n =$ the magic constant in a normal $n \times n$ magic square

6. the 179th term in the sequence $a_n =$(the sum of the first n^2 positive perfect squares)/n

7. the first three non-zero terms of the sequence $a_n = b_n - c_n$, where $b_n = 2^{n-1}$ and $c_n =$ the maximum number of non-overlapping regions into which the interior of a circle is divided if n distinct points on the circle are joined by all possible chords with no three chords intersecting at one point

8. the last term in the finite sequence $a_{n+1} =$ the minimum integer in the list $\{A(a_n, a_{n-1}), G(a_n, a_{n-1}), H(a_n, a_{n-1})\}$ (if such an integer exists) for $n \geq 2$, with $a_1 = 72$ and $a_2 = 18$, where $A(x, y) =$ the arithmetic mean of x and y, $G(x, y) =$ the geometric mean of x and y, and $H(x, y) =$ the harmonic mean of x and y

9. A look-and-say sequence is a sequence beginning with a single digit and the next term is obtained by describing the previous term. For example, the look-and-say sequence beginning with $b_1 = 1$ has second term $b_2 = 11$ since b_1 had "one one". The third term of this sequence would then be $b_3 = 21$ since b_2 had "two ones", and so on. Determine the 9th term of the look-and-say sequence beginning with $a_1 = 4$.

10. the second term in the increasing sequence $a_n =$ the length of the hypotenuse in a Pythagorean triple whose legs differ in length by 2

11. the seventh term in the ordinal sequence $\varnothing$, $\{\varnothing\}$, $\{\varnothing, \{\varnothing\}\}$, $\{\varnothing, \{\varnothing\}, \{\varnothing, \{\varnothing\}\}\}$, ...

12. Caroline has 62 blue balls and $3k-2$ red balls, where $k \geq 1$, to place evenly in three buckets ($k+20$ balls in each bucket). Let $a_{k,n} =$ the probability that exactly n red balls end up in a single bucket on a total of $3k-2$ red balls. Find the value of $a_{4,6}$, written as a fraction whose numerator and denominator are relatively prime.

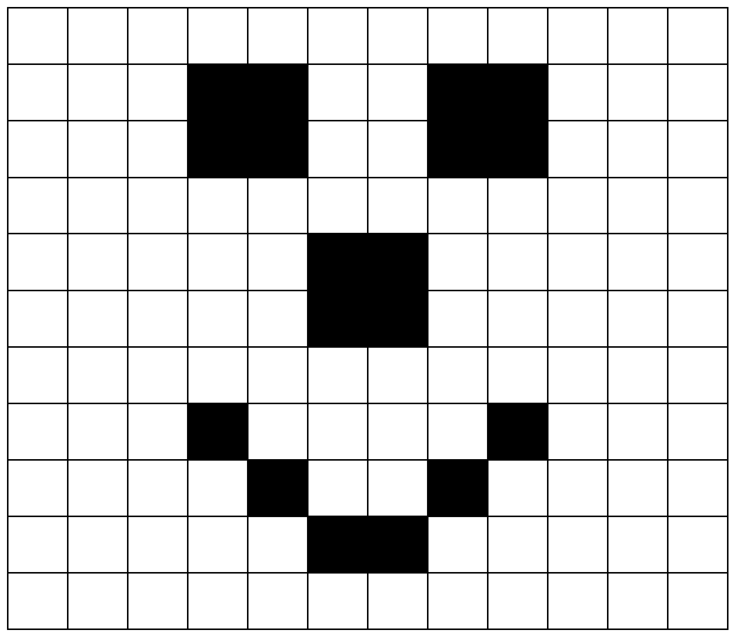
Section X – Life

For this question, we will play Life. The universe in the game is an infinite two-dimensional orthogonal grid of square cells, each of which is in one of two possible states: live or dead. Live cells are black and dead cells are white. For each stage, every cell interacts with its eight neighboring cells according to the following rules:

a) Any live cell with fewer than two live neighbors dies (underpopulation)

b) Any live cell with more than three live neighbors dies (overpopulation)
c) Any live cell with two or three live neighbors lives to the next stage
d) Any dead cell with exactly three live neighbors becomes a live cell in the next stage

Determine the still life final stage of the following pattern, which stays entirely within the 12×11 grid. Make sure that your answer is not only the correct shape but also in the correct blocks on the answer sheet (the original pattern will be shaded a lighter color on the answer sheet). 50 points for a correctly-formed final stage.



Section XI – Homeomorphism

In topology, a homeomorphism is a continuous function from one set onto another. To a topologist, two homeomorphic sets are indistinguishable. Two letters are homeomorphic if one can bend, but not break (separate into disconnected pieces), one letter to form the other. Clearly, homeomorphism of letters defines a partition. Determine which letters in the alphabet below are homeomorphic by partitioning them into sets such that within a single set, the letters are all homeomorphic. To get you started, C and S are homeomorphic because both can be straightened out into a single line. 5 points for each correctly formed homeomorphic set of the partition.

A B C D E F G H I J K L M

N O P Q R S T U V W X Y Z

Section XII – X-treme Sudoku

Fill out the Sudoku below in the standard way (each row, column, and smaller, bold-outlined 4×4 square must contain each of the integers from 1 to 16), with the added criterion that each of the two diagonals must also contain each of the integers from 1 to 16 (those diagonals are shaded for your convenience). 1 point for each correct white cell, 2 points for each correct shaded cell. For each incorrect cell, you will lose twice that number of points (-2 for white cells, -4 for each shaded cells).

		13		10	5	9			7	1				8	
		15	1					16			12	6			9
16	10			14					4	3				12	
	8				12	15			5				16	11	3
6						14	3		16	13		2	4		12
		14					10	9	6		4	5		3	
	15		8		7		4	2		5		16			
	4	10	16	2						8			1		
		12			16						6	8	7	2	
			4		14		2	13		9		11		15	
	6		13	15		5	9	7					3		
14		11	15		10	7		3	8						1
12	7	5				11			2	6				14	
	13				2	3					8			4	11
4			11	9			15					7	10		
	1				4	10			15	16	13		8		